## IMPLEMENTATION OF A SYSTEM OF FORCES EQUIVALENT TO ZERO IN MECHANICAL STRUCTURES

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## ABSTRACT

Determination of the internal forces and displacements of various sections of engineering structures are one of the most important tasks of a construction specialist. In this field of study, there exist several methods. These methods are used to calculate the displacements of a variable structure section, but the method described in this article gives a faster and easier way of defining the displacements of a variable structure section. This is a method that can be used by every engineer or other specialties of this area of study. This new method, derived and proved by restructuring and analysing traditional methods, allows us to define each structure section's linear displacements and rotation angles more simply. In this case, Mohr's graph doesn't need to find the reactions and construct the graphs of the internal forces.

Keywords: mechanical structures, displacement, system of forces equilibrated to zero, linear displacement

## **1 INTRODUCTION**

Several methods are developed over the years for the sole purpose of determining the displacements in different engineering structures. One of these traditional methods is the principle of virtual displacements. The method of virtual displacements is based on the principle that the work of the external and internal forces during a virtual displacement of a mechanical system in equilibrium is equal to zero [1-5]. Mohr integral also uses the same principle, but in contrast to the principle of virtual displacements, the displacements are taken from the real mechanical system. The forces, which in a mechanical structure are both internal and external, this method takes them from an imaginary system. This imaginary system is the Mohr system, where the structures is loaded with unit

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forces according to the direction of the displacements [6-11] we need to determine. In this article we are developing an alternative method to determine the displacement in engineering structure. This method is the method of forces equivalent to zero. Structure can be calculated in static and dynamic loads, but in this article, we are considering only the load that moves through the structure (perpendicular to the structure). We are not considering the dynamic effect of the forces. This method can be used in different fields of engineering and structural engineering models. Examples in this article are simple structure but this method also can be used in complex structure such as bridge, complex trusses etc. The method of force equivalent to zero, contrary to the Mohr method where we load the imaginary system with unit forces, we load the imaginary system with a system of forces equivalent to zero. The sign agreement is derived from the concept work, where the work is positive when the forces and the displacements are in the same orientation and negative when they are in the opposite direction. Equation 1, shown the principle of application of Mohr Integral, to find the displacement at point B in horizontal direction.

$$(1N) \cdot \delta_B^h = \sum_{i=1}^n \int_0^l \overline{M_{pk}} \cdot d\theta \to \delta_B^h = \sum_{i=1}^n \int_0^l \overline{M_{pk}} \cdot \left(\frac{M_p \cdot dx}{El}\right)$$
(1)

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Figure 1 Schematic drawing of the application of Mohr' integral with both the real and the imaginary drawing.

## 2 SYSTEM OF FORCES EQUIVALENT TO ZERO APPLIED IN AN ELEMENT OF A STRUCTURE

Equilibrated forces are a system of forces that are equivalent to zero [8, 9]. Such forces are plenty, but the most used forces for an element in a structure are shown in Figure 2, 3 and 4. Also, in these figures, is shown the case where a unit force is loaded, because the number value of 1 has the attribute that everything that is multiplied by it, doesn't change its value, unit forces are broadly used in structures. If we assume the forces equal to a unit of one, same as in Mohr integral, the following external forces systems are derived, with these corresponding self-equilibrated internal forces, which are easier to use [5, 9]. Figure 2 show a system of force equivalent to zero and in this case a system of two moments equal to zero.



Figure 2 Schematic drawing of two moments equivalent to zero and their relevant internal forces graph.

In figure 3 there is a couple of force and a moment that make this system equivalent to zero.



Figure 3 Schematic drawing of a couple force and moment equivalent to zero and the their relevant internal force.

And the last one is a system of force equivalent to zero, applied form section A to B, shown in figure 4. These systems of forces equivalent to zero have the characteristic that they affect only the part of the structure or system where they are applied. Thus, if we were to apply one of the systems shown in figures 2, 3 and 4, then the internal forces factors will occur only in this segment. In all the cases shown in this paper, we study only the bending moment of all the internal force factors, because the bending moment has the biggest influence percentage.



Figure 4 System of forces equivalent to zero and their relevant internal force.

All the actions described above, with the system of forces equivalent to zero, are done using the imaginary system. We apply all these actions in the imaginary schematic drawing shown in figure 5.



Figure 5 Left shows a simple structure with a system of two forces and one moment equivalent to zero; Right part shows the respective internal moment.

## 3 MOHR INTEGRAL SOLUTION FOR SOME SYSTEMS OF FORCES EQUIVALENT TO ZERO

Below, we will show some of the most used cases for calculating the internal forces factors. These cases serve as a basis for solving more complicated engineering structures.

#### 3.1 CASE 1

In the first case in figure 5, we determine the displacements. In the case shown in figure 6 we are determining the angle of rotation and for that reason the system of forces equivalent to zero the two-moment case (Figure 2) is used.



Figure 6 Left part shows a system of two moments equivalent to zero and the resulting internal moment; Right parts show the unknown displacements and the external forces.

The angle of rotation, which we need to determine in this case, is given by equation 2, which we can derive by applying the system of forces equivalent to zero with the two equilibrated moments.

$$1 \cdot \varphi_A + 1 \cdot \varphi_B = \int_0^{l_1} 1 \cdot \frac{M_p \cdot dx}{EI} = \frac{1}{EI} \int_0^{l_1} M_p \cdot dx = \frac{1}{EI} A_{A/B} (M_p)$$
(2)

Where,  $A_{A/B}(M_p)$  gives the area of moment  $M_p$  for the part from section A to section B.

#### 3.2 CASE 2

Even in case two, we have a relatively simple structure. We apply the system of forces equivalent to zero shown in figure 3, a force couple and a moment to achieve equivalence. Using the steps above we can now write the following equation to determine the general displacements, and more specifically, the angle of rotation  $\varphi_A$  and the displacements  $\delta_B$  and  $\delta_A$ .



Figure 7 Application of the system of forces equivalent to zero using the system given in figure3.

$$l_{1} \cdot \varphi_{A} + 1 \cdot \delta_{B} + 1 \cdot \delta_{A} = \int_{0}^{l_{1}} x \cdot \frac{M_{p} \cdot dx}{EI} = \frac{1}{EI} S^{B}{}_{A/B} (M_{p})$$
(3)

Where  $S^{B}_{A/B}(M_{p})$  is the static moment of the moment  $M_{p}$  form section A to B according to point B.

#### 3.3 CASE 3

By continouing with the same method and procedures as we have done above, we can continue with the third case where

we have three forces. Two forces are applied in the same orientation on the edges of the element and a third force applied arbitrary in between the two forces, in the opposite orientation, but with a value that makes the system equivalent to zero. We can achieve many similar situations with case 3 and for that reason we will show three subcaces, 3aI, 3aII and 3aIII, figure 8. Also, will show a special case where the force is applied in the middle. The case 3aI is achieved by applying two edge forces with value of 1/a and 1/b and the inbetween force of value (a+b)/ab, in figure 8. The displacements in this case can be calculated by the equation 4. Those displacements in this case are  $\delta_{forces}$ ,  $\delta^h_A$  and  $\delta^h_B$ , each corresponding to forces  $\frac{a+b}{ab}$ ,  $\frac{1}{a}$  and  $\frac{1}{b}$ , respectively.

$$\frac{a+b}{a\cdot b}\delta_{forces} + \frac{1}{a}\cdot\delta^{h}{}_{A} + \frac{1}{b}\cdot\delta^{h}{}_{B} = \int_{0}^{a}\frac{x}{a}\cdot\frac{M_{p}\cdot dx}{EI} + \int_{0}^{b}\frac{x}{b}\cdot\frac{M_{p}\cdot dx}{E$$

The same exact situation occurs if the forces on the edges are given as a and b and the in between force is a + b. Then, we can calculate this case by drawing the imaginary scheme and the real scheme as shown in 3aII. The displacements of case 3aII are given by equation 5.

$$(a+b)\delta_{forces} + b \cdot \delta^{h}{}_{A} + a \cdot \delta^{h}{}_{B} = b \int_{0}^{a} x \cdot \frac{M_{p} \cdot dx}{EI} + a \int_{0}^{b} x \cdot \frac{M_{p} \cdot dx}{EI} = b \frac{S^{A}{}_{A/force}(M_{p})}{EI} + a \frac{S^{B}{}_{B/force}(M_{p})}{EI}$$
(5)

Case 3aIII is achieved by giving the in between force a value of 1. The edge forces will be  $\frac{a}{a+b}$  and  $\frac{b}{a+b}$ . Figure 8, 3aIII shows the application of the system of forces equivalent to zero method for this case. Thus, the displacements are given by equation 6.

$$\delta_{forces} + \frac{b}{a+b} \cdot \delta^{h}{}_{A} + \frac{a}{a+b} \cdot \delta^{h}{}_{B} = \frac{b}{a+b} \int_{0}^{a} x \cdot \frac{M_{p} \cdot dx}{EI} + \frac{a}{a+b} \int_{0}^{b} x \cdot \frac{M_{p} \cdot dx}{EI} = \frac{b}{a+b} \frac{S^{A}{}_{A/force}(M_{p})}{EI} + \frac{a}{a+b} \frac{S^{B}{}_{B/force}(M_{p})}{EI} +$$
(6)



Figure 8 The application of the system of forces equivalent to zero method according to case 3.



Figure 9 Special subcase of case 3, where inbetween force is applied in the middle.

By splitting case 3 in thre subcases, we can reduce calculation time by effectively choosing the best subcase. Lastly, the special subcase of case 3 is shown below. This case is achieved if the inbetween force is in the middle as shown in figure 10.

$$2 \cdot \delta_{Mid} + 1 \cdot \delta^h_B + 1 \cdot \delta^h_A = \int_0^a x \cdot \frac{M_p \cdot dx}{EI} + \int_0^a x \cdot \frac{M_p \cdot dx}{EI} + \int_0^a x \cdot \frac{M_p \cdot dx}{EI} + \frac{S^B_{B/Mid}(M_p)}{EI} + \frac{S^B_{B/Mid}(M_p)}{EI}$$

## 4 DIFFERENT APPLICATIONS WITH THE METHOD OF SYSTEM OF FORCES EQUIVALENT TO ZERO

Lets take a simple beam where a single vertical load is applied, as shown in figure 11. An engineer duty is to calculate the displacements and rotation angles of the structure, in this case a beam. By using the method of the system of forces equivalent to zero, we can calculate the displacemetns or rotation angles. We start by applying the second case described by this method. We apply the system of forces equivalent to zero from section A to B.



Figure 10 Simple application on a beam.

The given data for this study case are the length of the beam l = 5m and moment in section A, M = 5 Nm. In figure 12, are shown the moment graph from the system of forces equivalent to zero. By applying equation 3 we get the following:

$$5 \cdot \varphi_A + 1 \cdot 0 + 1 \cdot 0 = \frac{S^B_{A/B}(M_p)}{EI} = \frac{4 \cdot 24}{2EI} \left(\frac{4}{3} + 1\right) + \frac{1^2}{6EI} \left(2 \cdot 24\right) = \frac{120}{EI} \to \varphi_A = \frac{24}{EI}$$
(8)



Figure 11 First step of the method, calculation of rotation angle in section A.

Next, we need to find the rotation angle in section B. For this reason, we apply the first case of the method as shown in figure 13. The results from the first step, where we calculated the rotation angle in section A, are very important for the continuity of this method. By using equation 2, we derive the following equation 9.



Figure 12 Applied case 1 to the given structure.

$$1 \cdot \varphi_A + 1 \cdot \varphi_B = \frac{A_A(M_p)}{EI} = \frac{24 \cdot 5}{2EI} = \frac{60}{EI} \rightarrow \varphi_B = \frac{60}{EI} - \tag{9}$$
$$\frac{24}{EI} = \frac{36}{EI}$$

If a displacement at an arbitrary section, for example section C of the beam, needs to be calculated, then we can continue by applying the case 3aII, because the load is not in the middle, as shown in figure 14.



Figure 13 Case 3aII of the method applied in a simply supported beam.

By applying equations 4-6, we getr the following equations:

$$5\delta_{forces} = 1 \frac{\sum_{I=1}^{S^{A}} (M_{p})}{\sum_{I}} + 4 \frac{\sum_{I=1}^{S^{B}} (M_{p})}{\sum_{I}} = \frac{4^{2}}{6EI} 2 \cdot 24 = \frac{160}{EI} \rightarrow \delta_{forces} = \frac{32}{EI}$$
(10)

Maximum displacement is supposed to occur in section D, between section A and C. In section D, the displacement is zero because we have an extremum and the first derivate gives the angle. To find the placement of section D, the first case of the system of forces equilibrated to zero is applied, by placing it in section A and D. The following equation is derived using equation 2.

$$1 \cdot \varphi_A + 1 \cdot \varphi_D = \frac{\frac{A_A(M_p)}{D}}{\frac{D}{EI}} = \frac{6 \cdot x_D \cdot x_D}{2EI} \rightarrow \frac{24}{EI} + 0 =$$
(11)  
$$\frac{3 \cdot (x_D)^2}{EI} \rightarrow x_D = 2\sqrt{2} = 2.8m$$

Where the moment is  $M_D = 6 \cdot x_D$ 



Figure 14 Application of case 1 for finding the section D placement in the beam.

To calculate the maximum displacement, we can load the beam with case 2 of the method from section A to B. Then we continue by using equation 3 accordingly. The known data are as follows,  $x_D = 2\sqrt{2}m$  and  $M = 2\sqrt{2}Nm$ , figure 16.



Figure 15 Application of case 2 in the simply supported beam.

The following equation is derived from the above argumentation.

$$2\sqrt{2} \cdot 0 + 1 \cdot 0 + 1 \cdot \delta_{max} = \frac{S^A A(M_p)}{E_I} = \frac{(2\sqrt{2})^2}{6E_I} \cdot 2 \cdot 6 \cdot (12)$$
$$2\sqrt{2} \to \delta_{max} = \frac{32\sqrt{2}}{E_I}$$

## 5 SYSTEM OF FORCES EQUIVALENT TO ZERO IN A STATICALLY DETERMINE AND INDETERMINATE STRUCTURE

This system of forces equivalents to zero, figure 2, 3 and 4, are also applicable in structures [12-16]. The structure can be determinate or indeterminate. If load a structure with the above systems, then as previously explained, only the part of the structure that is loaded with the system of forces equivalent to zero will produce internal bending moment [17-20]. Until known we have only used this method on statically determinate systems, but statically indeterminate systems can also be solved using the method this paper describes and proves. In the case of a statically indeterminate system, we have more mechanical linkages than needed, thus we have more sections with zero displacements, but on the contrary, we have more unknown values than needed. That is why the classic route is followed, where the main system of the force method is created first, then the moment diagrams are created separately for more kinematic parameters with values of zero, and at the same time, the extra reactions and kinematic parameters of the system are determined.

# 5.1 APPLICATION FOR INDETERMINATE SYSTEMS

Below, we will applicate this method described by this paper in some indeterminate systems. The first case consists of a simple beam that has a fixed support on section A and a vertical sliding sleeve.



Figure 16 First application of a system of forces equivalent to zero in an indeterminate system.

We smartly choose one of the systems equilibrated to zero, in this case a system of two moments in equilibrium applied from section A to B.

$$1 \cdot 0 + 1 \cdot 0 = \frac{A_A(M_p)}{EI} = \left[\frac{Pa \cdot a}{2EI} - \frac{M_B \cdot (a+b)}{EI}\right] \to Pa^2 -$$
(13)  
$$2M_B(a+b) = 0 \to M_B = \frac{Pa^2}{2(a+b)}$$

As given by the above calculations, we can find the unknown moment. At this point, we can continue solving the problem as a statically determinate problem. Thus, we apply the second case of this method from section A to C. It is the duty of an engineer to choose which case smartly and precisely and where the case should be applied on the structure.



Figure 17 Second application of a system of forces equivalent to zero.

$$a \cdot 0 + 1 \cdot 0 + 1 \cdot \delta_P = \frac{S^C C(M_P)}{EI} = \left[\frac{Pa \cdot a}{2EI} \cdot \frac{2}{3}a - \frac{Pa^2 \cdot a}{2EI \cdot (a+b)} \cdot \frac{a}{2}\right] = \frac{P \cdot a^3}{12EI \cdot (a+b)} (4(a+b) - 3a)$$
(14)

With the above calculations we can calculate the displacements needed.

## 6 CONCLUSIONS

From the study of structures subjected to this system of forces equivalent to zero, we have drawn the following main conclusions.

- Systems of forces equivalent to zero are a system that should be applied only between the sections that we want to do calculations.
- This method does not require to calculate the reactions of the supports, because the system itself is equivalent to zero.
- We have internal forces only in the part of the system where forces equilibrated to zero are applied.
- By applying the system of forces equilibrated to zero in different parts of the structure, we can determine the displacements of the entire structure.
- Knowing the allowed displacements of the different mechanical linkages of a structure, we can use them to reduce the number of unknown displacements.
- This method can solve statically determine and indeterminate systems easily, because we have many linkages, and we know a lot of displacements.

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