ISSN 1590-8844
International Journal of Mechanics and Control, Vol. 25, No. 01, 2024
https://doi.org/10.69076/jomac.2024.0009

ANALYSIS AND OPTIMIZATION MODELS FOR PUBLIC TRANSPORT SCHEDULING OF MEDIUM TO BIG CITIES

Arlinda Alimehaj Rrecaj, Ramadan Duraku

University of Pristina, Faculty of Mechanical Engineering, Traffic and Transport Department, 10000 Pristina, Kosovo

ABSTRACT

Urban public transport is a vital part of urban mobility, especially in cities with a significant young population. The appropriate public transport services should be seen from a number of aspects such as those in the context of fulfilling the passenger demands, as well as from that of the general costs, whether they are experienced by the non-transportation community-externalities. The fulfilment of the passenger's demand by increasing the scheduled number of bus trips in one side can result in increased fuel costs and air pollution on other side. In this paper, the trip frequency of the public transport lines of the city of Pristina has been analysed and improved by linear (LP) and non-linear programming (NLP) optimization methods according to the minimization of passenger waiting times at the stops of each route, respecting the constraints of the passenger demand, number of bus fleets, the travel time and the daily fuel budget.

Keywords: Trip frequency, NLP optimisation, waiting times, inter-arrival time, route.

1 INTRODUCTION

Optimization methods with LP and NLP programming are known for their great use in various fields of life where it is required to attack some quality measurement parameters and the fulfilment of different criteria. We have known them since the beginning of nutritionist for balancing the nutritional values of proper daily diets, optimizing financial plans, investment plans, distribution of goods, project management, logistics, health and in the field of control and management in general. The linear programming method turns out to be the most used in the optimization of the number of public transport trips to date, since the optimization process contains very simple linear functions that are related to transport requests or transport time through the length of certain routes. Berhan *et al.* in their research [1] developed an optimum bus assignment method using LP and then validated the model output with the performances of existing systems. Wicaksono and Harahap in their optimisation model [2] for determination of minimized number of trips, used two types of buses of different sizes for different shifts. The main constraint involved on the model use is the maximum trip a bus can make on routes for given shift. Addressing of the uncertainty of traffic flow volumes during time of day is evident in many research with LP in order to keep to the planned schedule during peak hour with unchanged headway and increased transport capacity of transportation vehicles. Simeunovic *et al.* in their paper [3] took into consideration uncertain time disparity of the change of traffic flow parameters, which enables the determination of frequency of public transport trips and maintain the satisfied level of service offered to users. A mathematical formulation model for school bus routing with the aim of optimization of students spent time on bus stops was developed by Manumbu, Mujuni and Kuznetsov in [4] where is assumed that each bus has fixed pick up points. The above model paid significant attention to the waiting times of users but neglected the overall costs deriving from the number of available bus fleet. Following the LP based models regarding trip frequency optimization of public transport we can encounter efforts of using linear goal programming methods. Tekin *et al*. in [5], [27] formulated a multi-objective optimization algorithm to obtain optimum number of trips with cyclic rotation of buses with the aim of minimization of operational costs. The model is based on a daily three-hour period with constrained number of buses and collected data by automatic bus fare collection through selected routes of Antalya city.

Contact author: Ramadan Duraku

Address: 10000 Pristina, Kosovo.
E-mail: ramadan.duraku@uni-pr.edu

The constraints of above the model are trip constraint, capacity constraint, demand constraint and length of route constraint. In all the above reviewed scientific works the constituent functions such as objective function and constraint equation seems to be linear and consequently such has been the optimization method.

A more complex model toward optimization of public transport schedule using NLP was carried out by Kornfeld and Resnikoff in [6], [30] and [31]. The data of passenger demand are assumed to pursue the Poisson process with exponential inter-arrival times, which directly reflect the nonlinearity of the objective minimize function. Parbo et al. used a bi-level minimization problem that is difficult to solve mathematically because of the non-linear convex problem of the passenger flow data first obtained by a non-linear mapping route choice model [7]. As seen from the existing literature, the NLP method is used effectively in public transport scheduling.

Within the scope of this study the optimum trip frequencies are assigned for the urban network of the city of Pristina that is comprised of 15 (fifteen) routes. The rest of the paper is comprised of the below units. Unit two briefly presents the methodology of model's creation which proceeds with the estimation of travel time of each route that are important in estimating the number of required trips for each route.

The LP and NLP models are given in units 2.1 and 2.2, respectively. The comparison of model results with explication is given in unit 3 and conclusion for further research. Within the last unit 4 chapter with the conclusions for further studies are given.

2 METHODOLOGY

The data used in the optimization models were collected in the field for each station of each line by traffic engineering researchers. Moreover, based on surveys of public transport users, it is also possible to draw up an origin-destination matrix for each sub-area of the urban area and each subjected route. As can be seen from figure 1, the network of urban public transport of Pristina is comprised of 15 main routes, and their dermal extension is seen to be towards the center, so called overlapping or superimposed lines, but despite this fact at this part of study this is out of scope of the paper.

2.1. TRAVEL TIME AND VELOCITY ESTIMATION

The basic data that are necessary for the design of the model, in addition to the passenger demand, such as route names, route lengths, number of stations on both directions are shown in the following Table I.

As it is stated at the introduction unit, the travel time is an important parameter required for the needs of the model that directly depends on transport vehicle velocity on the line. Moreover, travel time does not provide only the time during the movement of public transport vehicle-bus.

It contains the dwell time which presents the time of bus at the stops for exchange the passengers.

The main travel time components for each route are summarized in Table II [8] and [9].

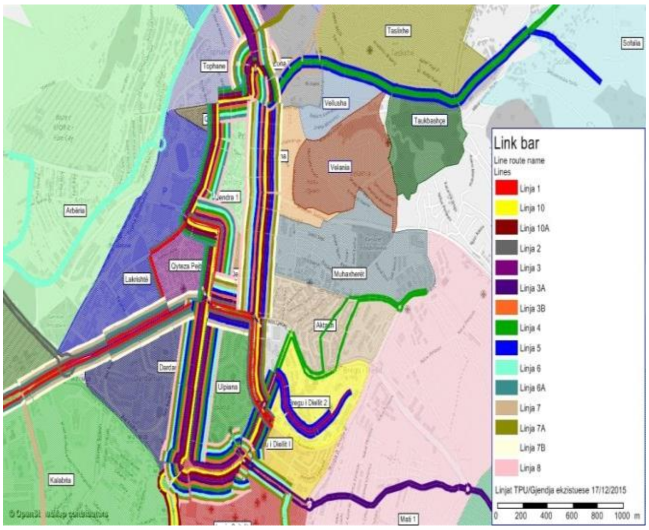


Figure 1 Map of existing urban bus routes of Pristina [12]

Table I - Basic data for the routes

Routes	Nr. of passengers	Route length [km]	Nr. of stations-direction A	Nr. of stations-direction B
Route 1	367	8.4	13	13
Route 2	262	13.7	13	15
Route 3	375	7.21	16	16
Route 3A	171	7.3	15	15
Route 3B	363	9.89	13	15
Route 4	143	8.74	21	20
Route 5	55	7.3	14	15
Route 6	58	8.67	19	20
Route 6A	208	10.72	14	14
Route 7	107	7.805	14	14
Route 7A	329	7.4	17	17
Route 7B	472	10.23	15	15
Route 8	82	21.5	17	16
Route 10	970	10.4	20	21
Route10A	156	8.71	10	12

The estimated travel time, T_t, is the sum of dw time, t_s, running time, t_r, and delay at signalized nodes, d, (1).

$$T_t = t_s + t_r + d \; [min] \tag{1}$$

$$t_s = t_b + t_{od} + t_{AB} + t_{cd} + t_a \; [min] \tag{2}$$

By substituting (2) in (1) is obtained the travel time expression in (3):

$$T_t = t_b + t_{od} + t_{AB} + t_{cd} + t_a + t_r + d \; [min] \tag{3}$$

Where are:

t_b	deceleration time at bus stop [s]
t_s	dwell time [s]
t_{od}	time of bus door opening [s]
t_{cd}	time of bus door closing [s]
t_{AB}	time of alighting and boarding of passengers [s]
t_a	acceleration time from bus stop [s]
t_r	running time [min]

Since we have the field data for each of the time parameters noted above, the following table has calculated the general travel time for each line.

Table II - Estimation of travel time Tt [min] for each route

Number of nodes	$t_b[s]$	$t_{od}[s]$	$t_{AB}[s]$	t_{cd} $[s]$	t_a $[s]$	$t_s[min]$	t_r $[min]$	Delay at intersections [s]	Estimated travel time *Tt* [min]	Ratio of Travel time to 60 min	Estimated velocity *V* [km/h]
7.0	3.5	2.0	12.8	2.0	2.0	0.4	12.6	3.5	32.9	0.549	30.6
6.0	3.5	2.0	9.2	2.0	2.0	0.3	20.6	3.0	47.7	0.795	34.4
8.0	3.5	2.0	13.1	2.0	2.0	0.4	10.8	4.0	30.4	0.506	28.5
5.0	3.5	2.0	6.0	2.0	2.0	0.3	11.0	2.5	27.4	0.457	32.0
9.0	3.5	2.0	12.7	2.0	2.0	0.4	14.8	4.5	39.4	0.657	30.1
4.0	3.5	2.0	5.0	2.0	2.0	0.2	13.1	2.0	30.7	0.512	34.2
8.0	3.5	2.0	1.9	2.0	2.0	0.2	11.0	4.0	30.3	0.505	28.9
5.0	3.5	2.0	2.0	2.0	2.0	0.2	13.0	2.5	31.4	0.523	33.1
8.0	3.5	2.0	7.3	2.0	2.0	0.3	16.1	4.0	40.7	0.679	31.6
4.0	3.5	2.0	3.7	2.0	2.0	0.2	11.7	2.0	27.9	0.464	33.6
5.0	3.5	2.0	11.5	2.0	2.0	0.4	11.1	2.5	27.9	0.465	31.8
7.0	3.5	2.0	16.5	2.0	2.0	0.4	15.3	3.5	38.6	0.643	31.8
4.0	3.5	2.0	2.9	2.0	2.0	0.2	32.3	2.0	68.9	1.149	37.4
8.0	3.5	2.0	34.0	2.0	2.0	0.7	15.6	4.0	40.6	0.677	30.7
6.0	3.5	2.0	5.5	2.0	2.0	0.2	13.1	3.0	32.6	0.544	32.0

The field data such as the length of the route, the number of signalized and non-signalized intersections included within the routes, passenger demand was made from the recordings of the students in the field. An appropriation of the parameters is made from the manual bus transit capacity [8] and Highway Capacity Manual (HCM) [9]. The travel time ratio represents the journey time of bus in each route within the chosen period. In below figure is graphically presented the journey travel time in minutes and velocity in km per hour.

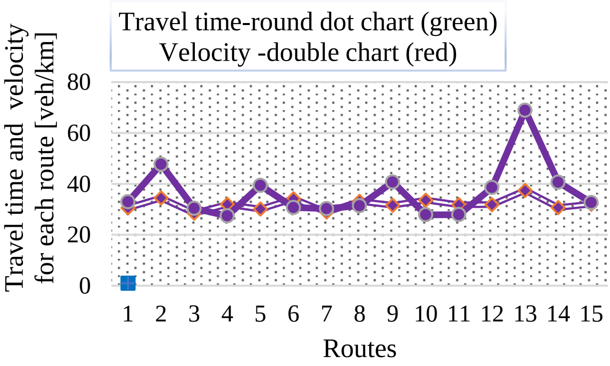


Figure 2 Travel time and velocity of each route

2.1. LP MODEL 1

Before giving the main elements of the model formulation, namely, the parameters, the number of variables, the objective function and the constraints, some general statements that hold for both models are given. In this paper chosen period is one 60 minutes a cycle time for each route. Since the variability of the passenger demand and traffic flow during the day is evident, we have distinguished the complete daily period through 9 periods known as shifts, where in 2 of them passenger demand varies. This means that within 18 daily hours, 4 hours are characterized by greater flows of passengers through the stations. Two of these shifts' present higher values of passenger demand, therefore the frequency calculations of buses on the routes are higher in these periods. The reason for this division is to determine the total number of trips for each line.

For example, if a bus trip is 20 minutes and the length of period is 4 hours this means that the required number of trips for this period is 12, consequently 3 trips for one shift period of 60 minutes. The creation of this model came because of the need to compile a schedule with a minimum number of buses, knowing that the available float of the public transport for Pristina is 52 buses, so the objective function is to minimize the number of buses per each route in cycle period T:

$$F_{min} = \sum_i^{15} X_i \cdot \frac{T_i}{T} \tag{4}$$

Where:

T_i	travel time for a route *i*,
T	cycle period time (60 minutes)

The capacity constraints are 16 in total:

$$X_i \cdot c_b \geq D_i; \tag{5}$$

$$X_{i+1} \cdot c_b \geq D_{i+1}; \tag{6}$$

$$\vdots$$

$$X_{15} \cdot c_b \geq D_{15};$$

$$\sum_{i=1}^{15} X_i \cdot c_b \geq \sum_{i=1}^{15} D_i \tag{7}$$

Where:

C_b	is the bus capacity (54 passengers)
D_i	passenger demand for route *i*.

Distance constraint:

$$\sum_{i=1}^{n} L_i \cdot X_i = L_{av} \cdot N_b \tag{8}$$

Where:

L_i	is the length of route i,
X_i	is the minimum number of trips required for route *i*,
L_{av}	is the average distance travelled by each bus on given routes, that is $(\sum L_i)/15$.
N_b	is the number of buses available.

The results are presented in Table III. Within the mathematical formulation, we notice that priority is given to respecting the total number of the bus fleet, but not the delays experienced by passengers through the stations of certain lines that represent the level of public transport service.

Table III - Results of number of buses for LP model

Travel time T_t for Route i		Number of trips (LP model)	Headways	Number of buses
Route 1	38	7	15	4
Route 2	42	5	20	3
Route 3	37	7	15	4
Route 3A	29	3	30	2
Route 3B	41	7	15	4
Route 4	30	3	60	1
Route 5	26	8	60	1
Route 6	27	1	60	1
Route 6A	37	4	30	2
Route 7	27	2	60	1
Route 7A	35	6	15	4
Route 7B	45	9	10	6
Route 8	47	2	30	2
Route 10	63	10	4	15
Route10A	31	3	30	2
				∑ 52

Also, the inclusion of different shifts has been neglected and with this only the values of the number of trips that are the same for each time cycle throughout the day have been obtained.

2.2 NLP MODEL 2

2.2.1 Poisson Distribution with Exponential Inter-Arrival Times of Passengers

The arrival of passengers at bus stops is considered as a Poisson process where their inter-arrival times are considered to pursue exponential distribution. We assume the arrival of buses in deterministic time *T* within this interval *T* passengers arrive at certain bus stop on arrival times *S*$_1$, *S*$_2$...*Sn* in [0, *T*], so the waiting time of passengers is $W_p = \Sigma(T-S_i)$. The *N(t)=n* is the number of arrivals in [0, t]. The total waiting time of passengers W_p [10,11] is:

$$W_p = T - S_1 + T - S_2 + T - S_3 \ldots + T - S_k = \\ = n \cdot T - \sum_{i=1}^{n} S_i \tag{9}$$

The expectation of waiting time after conditioning the N(t) is:

$$\{W_p | N(t) = n\} = E[nT - \sum_{i=1}^{n} S_i \,|\mathrm{N(T) = n}] = \\ = nT - \sum_{i=1}^{n} \frac{T}{2} = nT - \frac{nT}{2} \tag{10}$$

According to the property of the Poisson process [6], [12], the inter-arrival time of occurrence is $1/\lambda$, so we can assume that a passenger will arrive in every $1/\lambda$ minutes. After involving the inter arrival time λ the equation (10) and regulating it become as in (11):

$$E(W_p) = T \cdot \lambda T - E(\sum_{i=1}^{N(T)} ES_i) = \\ = \lambda \cdot T^2 - (\sum_{i=1}^{N(T)} i/\lambda) = \\ = \lambda \cdot T^2 - \frac{1}{\lambda} E\left(1 + 2 + \cdots + N(T)\right) = \\ = \lambda \cdot T^2 - \frac{1}{2\lambda}\Big(\big(1 + N(T)\big)N(T)\Big) = \\ = \lambda \cdot T^2 - \frac{1}{2\lambda}(\lambda T^2 + \lambda T^2 + (\lambda T^2)^2) = \\ = \lambda \cdot \frac{T^2}{2} - T \tag{11}$$

2.2 MODEL 2 FORMULATION

In our model the objective function is minimization of passengers waiting time on each bus stop waiting time. The optimization process is conducted in MATLAB live desk editor for each of routes, for nine shifts (different one-hour periods of days). We agree to denote the model decision variables with X_i and Y_i, where: *Xi* is the average bus inter arrival time for certain route period i while Y_i is the number of necessary buses for certain route during period *i*.

After the minimization function has been determined, the daily budget limitations for fuel consumption expenses are also laid out as well as the bus fleet limitations, which in this model we have increased to 80 buses for the whole public transport network [13],[14] and [15]. The variables of the constraints are:

T	is the chosen interval or meaningful cycle period (60 minutes)
pi	inter-arrival time of passengers in station during the period i
li	is the length in kilometers traveled by bus during one period i
Nb	number of bus fleet available,
fb	fuel budget (euro) per day for give route,
fe	fuel efficiency-run kilometers per fuel liter,
fp	price of fuel in euros per liter.
v	average velocity for given route (estimated in unit 2),
sh	number of shifts (nine shifts each two hours are 18 hours in total for a working day,
s	number of stations in certain route.

In this model, approximately equal passenger arrivals are assumed between stations, therefore passenger waiting times are multiplied by the number of component stations in a line. Relating to equation 11 in the previous unit for waiting times, we have the objective minimum function (12):

$$F_{min} = \sum_{i=1}^{s}(\sum_{i=1}^{sh} \frac{T}{X_i}(\frac{X_i^2}{2p_i} - X_i));\ i \in \{1, \ldots sh\} \tag{12}$$

Function (12) is optimized subject to the following constraints [17] and [18].
Constraint of bus fleet:

$$Y_i < N_b;\ \ i \in \{1, \ldots sh\} \tag{13}$$

Constraint of daily fuel budget:

$$\frac{l_T \cdot f_p}{f_e} \cdot \sum_{i=1}^{sh} Y_i \leq f_b\ ;\ i \in \{1, \ldots sh\} \tag{14}$$

Number of buses based on travel time:

$$Y_i = \frac{T \cdot T_t}{X_i}\ \ ;\ i \in \{1, \ldots sh\} \tag{15}$$

All variables are non-negative and $Y_i < X_i$. Before running the NLP model, the data to be used is previously prepared on Table IV.

3 RESULTS AND DISCUSSION

After we have presented the results of the two models carried out, the comparative results between these models are presented on Table V.

Table IV - Input data for NLP Model

Route	Length [km]	Fuel cost unit	No. of stations	Travel time ratio
Route 1	8.4	3.361	13	0.549
Route 2	13.7	4.992	13	0.795
Route 3	7.21	2.962	16	0.506
Route 3A	7.3	3.881	15	0.457
Route 3B	9.89	3.662	13	0.657
Route 4	8.74	4.540	12	0.512
Route 5	7.3	4.258	17	0.505
Route 6	8.67	4.933	15	0.523
Route 6A	10.72	4.474	14	0.679
Route 7	7.805	4.455	14	0.464
Route 7A	7.4	3.288	17	0.465
Route 7B	10.23	3.507	15	0.643
Route 8	21.5	7.095	17	1.149
Route 10	10.4	2.537	20	0.677
Route10A	8.71	4.319	10	0.544

Table V - Comparison of number of trips-buses

	LP Model		NLP Model	
Route	Inter-arrival time of bus	Number of buses	Inter-arrival time of bus	Number of buses
Route 1	15	4	7	6
Route 2	20	3	7	9
Route 3	15	4	5	8
Route 3A	30	2	5	3
Route 3B	15	4	10	8
Route 4	60	1	5	2
Route 5	60	1	15	1
Route 6	60	1	26	2
Route 6A	30	2	15	4
Route 7	60	1	10	3
Route 7A	15	4	10	4
Route 7B	10	6	10	5
Route 8	30	2	10	5
Route 10	4	15	15	13
Route 10A	30	2	10	4
Σ		**52**		**77**

It can easily be seen that for all the routes included in the optimization we have a difference in the results regarding the frequency of trips within the one-hour period. The increase in the fleet has enabled the frequency of trips to improve objective function value, reducing the waiting times of passengers through bus stations. The results are also presented graphically in figures 3 and 4. From the results we observe that the NLP model provided an efficient solution for determining the optimum number of trips and inter-arrival times of the buses. We verify the solution through the value of the objective function. The comparison of passenger waiting times across stations is given in table 6 and figure 5. In all routes there is a decrease in this parameter, which concludes that through our model with non-linear optimization it has been achieved to improve the level of public transport service in Prishtina. Since the first model, with linear optimization, does not consider the waiting time of the passengers, we used the platform of the non-linear model for the calculation of this quantity [25], [26], [27] and [28].

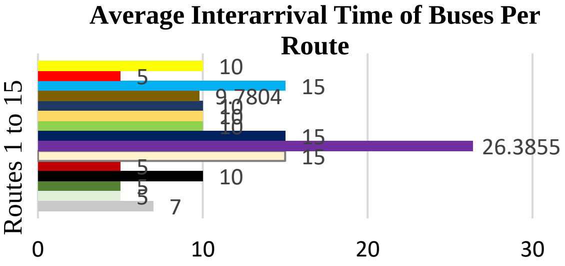


Figure 3 Average bus inter-arrival time.

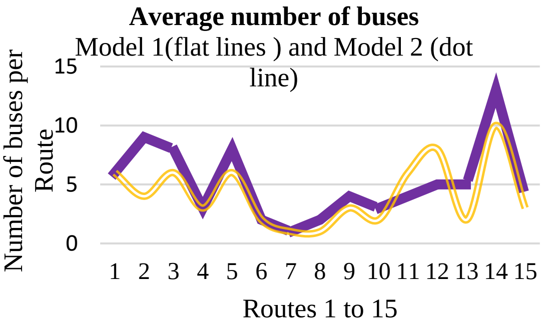


Figure 4 Comparison of number of trips-buses.

The process was carried out in such a way that the data obtained for the number of buses in the first model were entered as input data in the second model and we obtained the result of the number of passengers. in all lines, satisfactory results of the NLP model over the LP model have been obtained.

Table VI - Passenger waiting times [min]

	Total Waiting time of passengers NLP Model [min]	Total Waiting time of passengers LP [min]
Route 1	13009.4	19500
Route 2	726.29	18200
Route 3	10800	16000
Route 3A	8244.1	8730
Route 3B	1170	15560
Route 4	10530	9090
Route 5	6161.72	7236
Route 6	4699.3	10251
Route 6A	10500	12096
Route 7	3920	6378
Route 7A	17850	4590
Route 7B	13500	16200
Route 8	5911.4	7110
Route 10	36000	30000
Route 10A	6824	8150
	149846	**189091**

The difference in the total value of passenger waiting times between the two models is approximately 40 000 minutes. The maximal difference is obvious for Route 2, with respective cell highlighted with yellow colour. In Fig. 5 the same waiting times are graphically presented, where the curves with double lines are those of the LP method, while those with dashed lines of the NLP method.

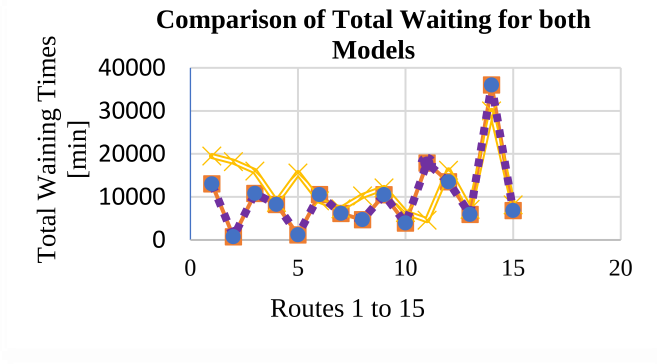
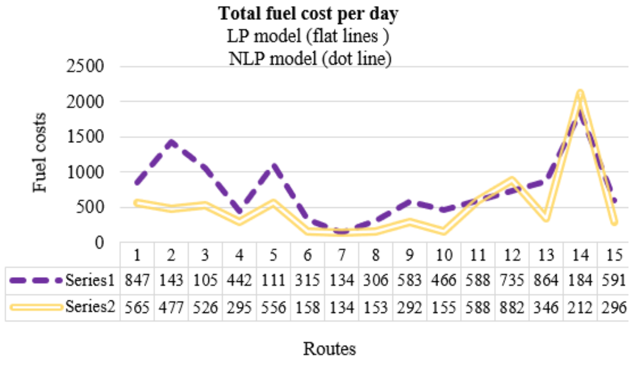


Figure 5 Passenger waiting times.

The function value of waiting times for each route is presented in the following Figure 6.

4 ANALYSIS OF FUEL COST AND SEEKING FOR FUTURE SOLUTIONS

In the comparative analysis of the models, the daily fuel expenses for the entire network are also included (figure 7) [17], [18] and [19]. It is obvious that high values have been obtained in the second model even though the number of passengers waiting times has been greatly reduced in this model. In fig. 7 represented the total daily fuel costs for each route.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Series1	847	143	105	442	111	315	134	306	583	466	588	735	864	184	591
Series2	565	477	526	295	556	158	134	153	292	155	588	882	346	212	296

Figure 7 Total fuel costs for each route.

The optimization of the frequency of trips has been made for the current configuration of the city's routes, as they are now organized [20], [21] and [23]. The analysis and the effort to reduce the cost should be seen from another point of view, such as that of the reorganization of the overlapped routes because they cause high costs, as the frequency of bus movements is greater. The current routes have many repetitions throughout the general network, so they have a pronounced overlap. We can see the ratio of repetition throughout the entire network in Figure 8. The majority of routes, such as routes: 10, 5, 6, 3A, 7B, 7, 7A take place simultaneously or pursue the 25 % of the entire network length. Routes 6, 5, 3A and 10 are provided in 60 % of the entire network length. Overlapping routes cause bus bunching. Bus bunching results in non-satisfied passengers feedback in aspect bus service and is one of the critical problems faced by transport community.

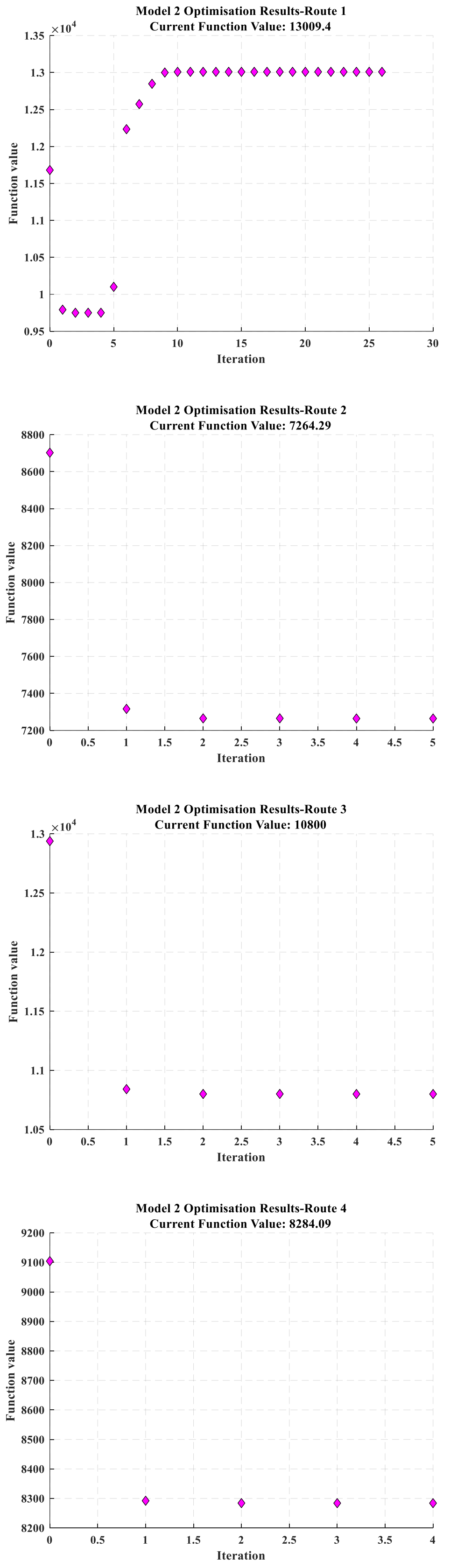


Figure 6 Results of objective function value for each route.

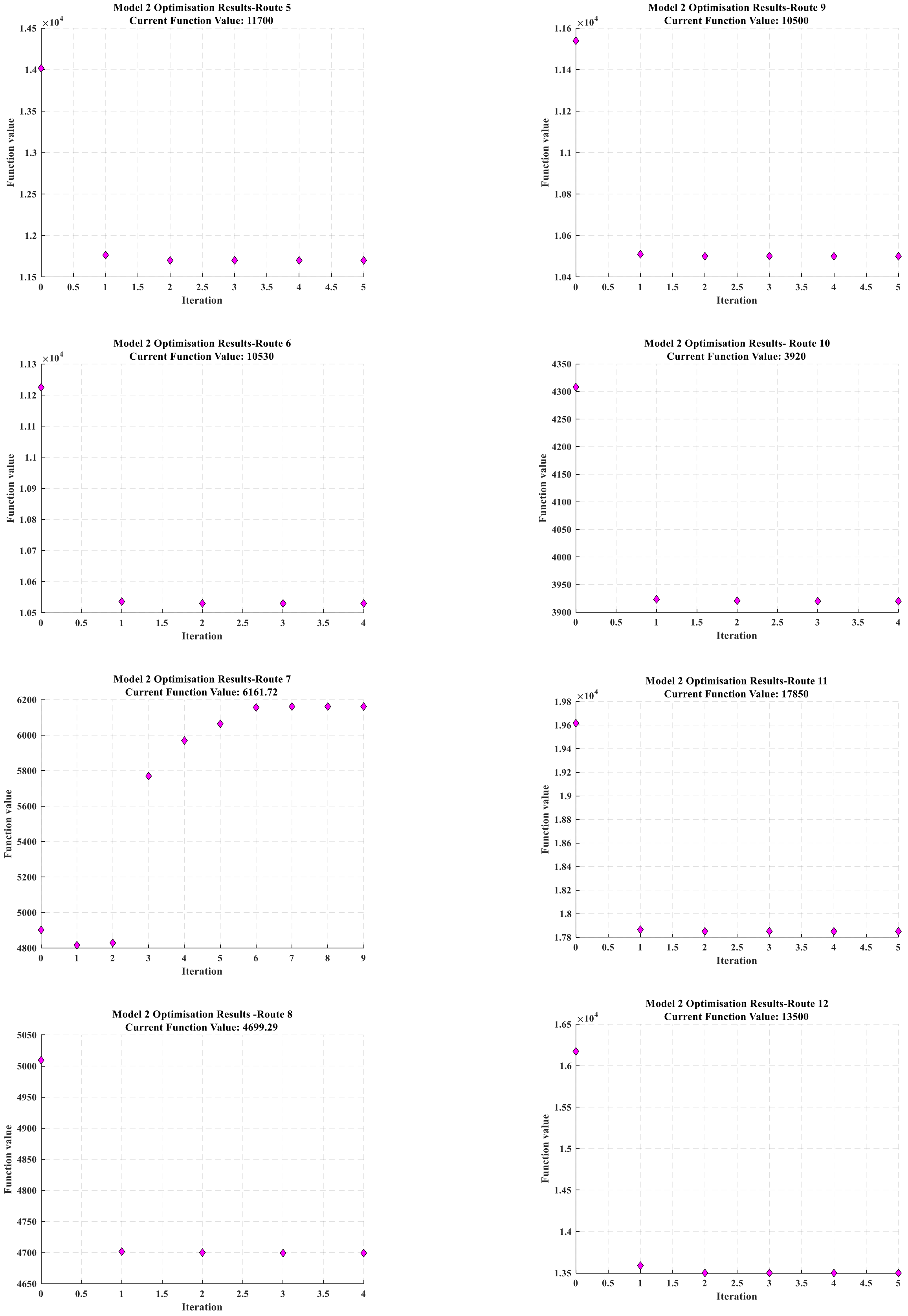


Figure 6 Results of objective function value for each route (continue).

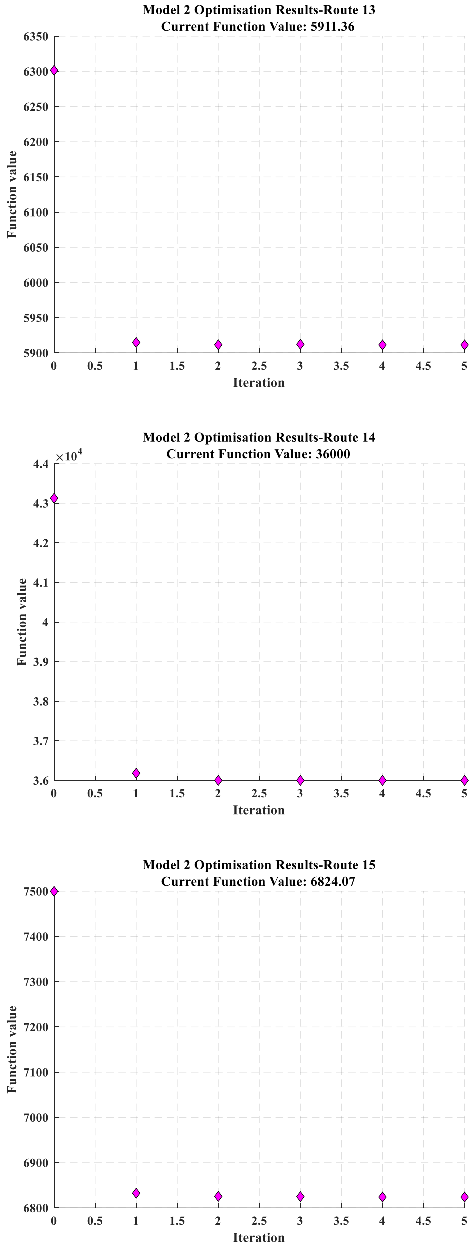
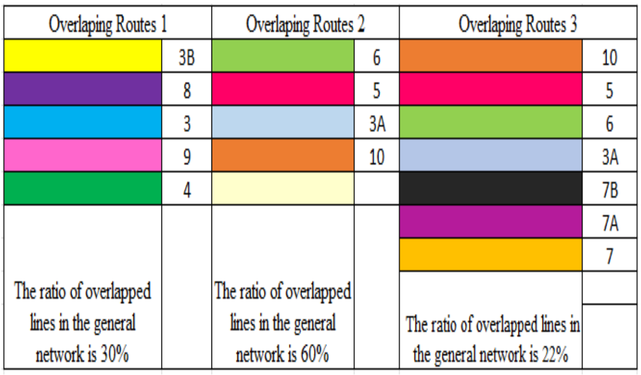


Figure 6 Results of objective function value for each route (continue).



Overlaping Routes 1		Overlaping Routes 2		Overlaping Routes 3	
🟨	3B	🟩	6	🟧	10
🟪	8	🟥	5	🟥	5
🟦	3	🟦	3A	🟩	6
🟪	9	🟧	10	🟦	3A
🟩	4	🟨		⬛	7B
The ratio of overlapped lines in the general network is 30%		The ratio of overlapped lines in the general network is 60%		🟪	7A
				🟨	7
				The ratio of overlapped lines in the general network is 22%	

Figure 8 The ratio of overlapping routes in network.

Another important aspect is the expected time that a passenger must wait before the arrival of its bus. The bunching phenomenon can be reduced by increasing the inter arrival time interval of buses or time elapse between two consecutive buses [18]. A reorganization of the routes means reducing them as well as reducing their overlap (Fig. 9).

The centre of the city would be facilitated by the creation of a central route in which other peripheral routes could be integrated. With this, the expenses will certainly be reduced [12, 24-26].

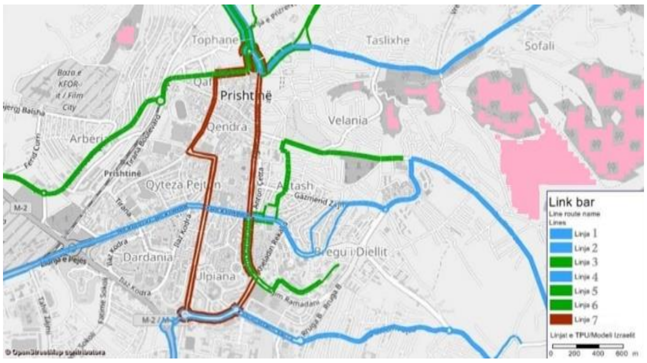


Figure 9 Proposed route reorganization [12].

5 CONCLUSIONS AND RECCOMENDATIONS

In this paper, the trip frequency of the public transport lines of the city of Pristina has been analysed and improved by linear (LP) and non-linear programming (NLP) optimization methods according to the minimization of passenger waiting times at the stops of each route, respecting the constraints of the passenger demand, number of bus fleets, the travel time and the daily fuel budget. In our NLP model the objective function is minimization of passengers waiting time on each bus stop waiting time. The optimization process is conducted in MATLAB live desk editor for each of routes, for nine shifts (different one-hour periods of days). Since the variability of the passenger demand and traffic flow during the day is evident, we have distinguished the complete daily period through 9 periods known as shifts, where in 2 of them passenger demand varies. In NLP model the arrival of passengers at bus stops is considered as a Poisson process where their inter-arrival times are considered to pursue exponential distribution. From the results we observe that the NLP model provided an efficient solution for determining the optimum number of trips and inter-arrival times of the buses. The difference in the total value of passenger waiting times between the two models is approximately 40 000 minutes. Like any model, the treated optimization model has some limitations in use. It should not be used without a prior verification of the nature of the distribution of passenger arrival times at the bus station. The treated public transport network is highlighted with overlapping lines, and even half of them pass through the middle district of the city. This overlap can cause congested traffic especially, simultaneous arrival of buses through stations of different lines, delays of buses on lines and dissatisfaction among passengers.

As the next step regarding the improvement would be to adjust the configuration of the lines. it would be more profitable for the number of lines to be reduced. to create a circular line in which the other lines that continue towards the suburbs can be integrated.

APPENDIX – MATLAB CODE FOR NLP MODEL

```
% Create optimization variables
a92 = optimvar("a","LowerBound",5,"UpperBound",60);
b91 = optimvar("b","LowerBound",5,"UpperBound",60);
c91 = optimvar("c","LowerBound",5,"UpperBound",60);
d91 = optimvar("d","LowerBound",5,"UpperBound",60);
e91 = optimvar("e","LowerBound",5,"UpperBound",60);
f91 = optimvar("f","LowerBound",5,"UpperBound",60);
g91 = optimvar("g","LowerBound",5,"UpperBound",60);
h91 = optimvar("h","LowerBound",5,"UpperBound",60);
i91 = optimvar("i","LowerBound",5,"UpperBound",60);
j91 = optimvar("j","LowerBound",1,"UpperBound",6);
k91 = optimvar("k","LowerBound",1,"UpperBound",6);
l91 = optimvar("l","LowerBound",1,"UpperBound",6);
m91 = optimvar("m","LowerBound",1,"UpperBound",6);
n91 = optimvar("n","LowerBound",1,"UpperBound",6);
o91 = optimvar("o","LowerBound",1,"UpperBound",6);
p91 = optimvar("p","LowerBound",1,"UpperBound",6);
q91 = optimvar("q","LowerBound",1,"UpperBound",6);
r91 = optimvar("r","LowerBound",1,"UpperBound",6);

% Define problem objective
problem.Objective = 15*60*[(a92/4-1)+(b91/4-1)+(c91/4-
1)+(d91/4-1)+(e91/4-1)+(f91/4-1)+(g91/4-1)+(h91/4-1)+(i91/4)];

% Define problem constraints
problem.Constraints.constraint1 =
4.3*(j91+k91+l91+m91+n91+o91+p91+q91+r91) <= 1000;
problem.Constraints.constraint2 = j91-60*(0.8)/a92 == 0;
problem.Constraints.constraint3 = k91-60*(0.8)/b91 == 0;
problem.Constraints.constraint4 = l91-60*(0.8)/c91 == 0;
problem.Constraints.constraint5 = m91-60*(0.8)/d91 == 0;
problem.Constraints.constraint6 = n91-60*(0.8)/e91 == 0;
problem.Constraints.constraint7 = o91-60*(0.8)/f91 == 0;
problem.Constraints.constraint8 = p91-60*(0.8)/g91 == 0;
problem.Constraints.constraint9 = q91-60*(0.8)/h91 == 0;
problem.Constraints.constraint10 = r91-60*(0.8)/i91 == 0;

% Set nondefault solver options
options91 = optimoptions("fmincon","PlotFcn","optimplotfval");
% Display problem information
show(problem);

% Solve problem
[solution,objectiveValue] =
solve(problem,initialPoint91,"Solver","fmincon",...
        "Options",options91);

% Display results
solution
objectiveValue

% Clear variables
clearvars a92 b91 c91 d91 e91 f91 g91 h91 i91 j91 k91 l91 m91
n91 o91 p91 q91 r91 initialPoint91 options91...
  objectiveValue
```

REFERENCES

[1] Berhan E., Mengistu D., Beshah B., and Kitaw D., Modeling and analysis of bus scheduling systems of urban public bus transport. *International Journal of Computer Information Systems and Industrial Management Applications*, Vol. 6, pp. 404-412, 2014.

[2] Wickanso Y. and Harahap M. Analysis of travel time and bus utilization of corridor 1 trans Jakarta to realize the smart mobility concept. *Journal Advances in Economics, Business and Management Research*, Vol. 72, pp. 345-352, 2018.

[3] Simeunovic M., Bogdanovic V., Simeunovic M., Pitka P., Papic Z. and Draskovic D., The model of the optimal number of public transport vehicles in mixed traffic flow conditions: A case study. *Journal of Discrete Dynamics in Nature and Society*, 2021.

[4] Manumbu D., Mujunu E. and Kuznetsov D., Mathematical formulation model for a school bus routing problem with small instance data. *Mathematical Theory and Modeling*, Vol. 4, No.8, pp. 121-134, 2014.

[5] Tekin S., Kofteci S., Aydin M. and Yildirim M., Trip optimization for public transport systems with linear goal programming (LPG) method. *Sigma Journal of Engineering and Natural Sciences*, Vol. 36, No.4, pp. 921-933, 2018.

[6] Kornfeld S., Ma W. and Resnikoff A., Optimizing bus schedule to minimize waiting time. *Operations Research II*, 21-393.

[7] Parbo J., Nielsen O. and Prato C., User perspectives in public transport timetable optimisation. *Transportation Research Part C*, Vol. 48, pp. 269-248, 2014.

[8] *Transit Capacity and Quality of Service Manual, Bus Transit capacity, Part 4* - 2nd Edition, pp.4-10, 2017.

[9] *Highway Capacity Manual*, 6th Edition, A guide for multimodal mobility analysis, Transportation Research Board, TRB, Washington. D.C.

[10] Esfeh M., Wirasinghe S., Saidi S. and Kattan L., Waiting time and headway modelling for urban transit systems- a critical review and proposed approach. *Transport Reviews*, 2020.

[11] Arhin S., Gatiba A. and Anderson M., Acceptable wait time models at transit bus stop. *Journal Engineering Technology and Transit Bus Stops*, Vol.9. No. 4, pp. 4574-4580, (2019).

[12] Sylejmani E., Analiza dhe modelimi transportit public në qytetin e Prishtinës, Universitety of Pristina, https://www.academia.edu/31504183/Public_transport_Prishtina

[12] Last G. and Penrose M., *Lectures on the Poisson process*. Cambridge University, 2017.

[13] Kockelman K., Chen D., Larsen K. and Nichols B., *The Economics of Transportation Systems: A reference for Practioners*. Amazon Group Publishing LLC, 2013.

[14] Immers L. and Stada J., *Basic Transport Economics*. Katholieke Universiteit of Leuven (Belgium), 2005.

[15] Li X., Yang Z. and Lian F., Optimizing on-demand bus services for remote areas. *Sustainability*, Vol 15, pp. 2-20, 2023.

[16] Seo J., Cho S., Kim D. and Park P., Analysis of overlapping origin-destination pairs between bus stations to enhance the efficiency of bus operations. *The institution of engineering and technology, IET Intelligent transport systems*, 2020.

[17] Koacer E., Koruca H. and Gamoura S., Evaluation of waiting time and transport costs with simulation in urban bus transport. *International journal of Applied Mathematics, Electronics and computers*, Vol.7, No. 4, pp. 84-87, 2019.

[18] Koppisetti M., Kavitha V. and Ayesta U., Bus schedule for optimal bus bunching and waiting times. *Institute of Electocal and Electronc Engineering*, 2019.

[19] de Vasconcelos N.V.C., Cavalcante C.A.V. and Palhares R.A., A new perspective for resolution forward reserve problem (Frp) through the goal programming. *International Journal of Mechanics and Control*, Vol. 22, No. 02, pp. 137-146, 2021.

[20] Petti G., Quattrocchi G. and Dalla Vedova M.D.L., Optimization methodologies study for the development of prognostic artificial neural network. *International Journal of Mechanics and Control*, Vol. 22, No. 01, pp. 3-9, 2021.

[21] Arena S., Orrù P.F., Perra F. and Pilloni M.T., Optimization of drugs delivery routes through location routing problem (Lrp): A case study. *International Journal of Mechanics and Control*, Vol. 22, No. 01, pp. 31-43, 2021.

[22] Valigi M.C., Logozzo S., Alvino A. and De Angelis F., Dynamic Characterization of Wire Rope Isolators for Transportation of Aerospace Electronic Systems. *International Journal of Mechanics and Control*, Vol. 22, No. 01, pp. 105-114, 2021.

[23] Spano M., Delvescovo D., Sangeorzan B., Misul D. and Belingardi G., Optimizing Engine Downsizing and Driving Behaviour in Conventional and Hybrid Powertrains for Autonomous Driving Applications. *International Journal of Mechanics and Control*, Vol. 21, No. 02, pp. 1-12, 2020.

[24] Jena S.R., Senapati A. and Geberemedhin G.S., Numerical Study of Solitions in Bfrk Scheme. *International Journal of Mechanics and Control*, Vol. 21, No. 02, pp. 163-175, 2020.

[25] Dežman M. and Gams A., Optimization and Analysis of the Modified Plvl-Variable Stiffness Actuator. *International Journal of Mechanics and Control*, Vol. 20, No. 01, pp. 23-33, 2019.

[26] Ricciu R., Besalduch L.A., Minisci E. and Manuello Bertetto A., Uncertainty Based Optimal Planning of Residential Building Stocks Retrofits. *International Journal of Mechanics and Control*, Vol. 20, No. 01, pp. 127-132, 2019.