CONVERSION OF PEAUCELLIER–LIPKIN STRAIGHT-LINE MECHANISM TO COMPACT COMPLIANT DEVICE

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ABSTRACT

Structures of straight-line mechanisms are very often used in building of smart precise robotic devices, to which requirements based on high accuracy and precision with manipulation of micro- and nano- scale objects are put. In this case only compact compliant devices can be used. The parasitic deformations in movement of compliant mechanisms must be minimized, when exact straight-line output motion is required. This put higher requirements to complexity of such devices design. This paper deals with design procedures (conversion) for device based on compact compliant mechanical structure, where kinematical structure of Peaucellier–Lipkin straight-line mechanisms was used. Design problems of transformation between straight-line mechanisms and flexure structures are discussed.

Keywords: compact compliant devices; precise positioning; Peaucellier–Lipkin mechanism; mechanisms design; modelling and simulation;

1 INTRODUCTION

The modularity of robotic devices is current trend in design of robotic devices [1, 2], but in the case of small robotic devices based on compact compliant mechanical structures [3, 4, 5] is this approach impossible. Such devices are usually build from one piece of material and output motion is produced by elastic deformation of material. In comparison with classic constructions of robot mechanisms such devices have one disadvantage and this is restricted movement of whole mechanisms, but this is the advantage of such devices too, in relationship to positioning accuracy. Compliant mechanisms can be divided in two groups: structures with concentrated (lumped) and distributed flexibility. In the first group of mechanisms the rigid parts/arms are connected by flexible joints. The second group, beside rigid parts, includes flexible joints and flexible arms too. In this paper the mechanisms with elastic joints only will be discussed.

The reason of choice of compliant mechanisms with elastic joints is similarity between design of classic construction of mechanisms and such devices [6, 7]. Main focus will be oriented on the plane mechanisms problems solution.

The design of compact compliant mechanical structure based on Peaucellier-Lipkin straight-line mechanism is the main aim of the paper. Design of precise positioning device with minimized parasitic deformations and with straightline output motion enables utilization of not so complex control system. In other words, because the mechanisms produce straight-line output motion with minimized parasitic deformations we shouldn't use the control system which needn't solve tasks connected with compensations of undesired motions. Other advantage of chosen approach is based on the fact, that usually only one actuator for control of one moving axis is used. In the case of flexure devices it is not attainable to use one actuator to one joint as it is in the classical kinematical structures.

2 STRAIGHT-LINE MECHANISMS

Mechanisms which are designed to produce a straight-line motion are known as straight line motion mechanisms [8]. Such mechanisms very often convert rotary motion to straight-line motion. A mechanism with sliding pair is the most common type of such devices, but sliding pair is

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bulky and subjected to comparatively rapid wear so a mechanism is constrained by the use of turning pair, or as combination of both types of kinematics pairs. According to [8] straight-line mechanisms are divided into three categories:

- Exact straight-line motion mechanism
- Approximate straight-line motion mechanism
- Straight-line copying mechanism

Extract straight-line motion mechanism follows a mathematical relation which holds true for all positions of input link such that output link follows straight-line [8]. The most known such devices are Peaucellier–Lipkin linkage, Hart mechanism and Scott Russell linkage. Only last mentioned device is composite from combination of sliding and turning pairs, other mechanisms are built only from turning pairs.

Some mechanisms describe an approximate straight-line motion; usually a four bar chain or its modified variant. To this category belong: Watt mechanism, Tchebicheff's mechanism (some resources talk about Chebyshev linkage), Grasshopper mechanism and Roberts's mechanism.

In design of precise micro-robotic devices are often utilized the straight-line kinematic structures. However complexity of such structures could not enable using of all structures in compliant mechanical devices. Therefore only kinematics of Hoeken lineage, Scott-Russell straight-line mechanisms and Watt mechanism are utilized in design process of small robotic mechanism.

2.1 MICRO-GRIPPERS UTILIZED STRAIGHT-LINE STRUCTURES

When is talked about gripping of small object is one important requirement connected by moving of gripper fingers. Many tasks require that movement of finger of gripper will be moving parallelly with all gripper's fingers [9, 10], in case of planar devices it mean movement of two fingers. But parallel movement of some fingers is possible to gain only by utilization of straight-line structure in structure of whole gripper. The design of griper with straight-line jaw motion of fingers (see Figure 1) may be desired for micromechanical tension/compression tests, and for gripping soft objects such as cells, gels, and assemblies of nanostructures such as carbon nano-tubes. This gripper utilizes Hoekens linkage with connected parallelogram for removing rotation of output point in respect to global coordinate system [10]. Similar approach for design of robot end-effector is presented on [11], where a modified Scott Russell straight-line mechanism was utilized.



Figure 1 Structure of compliant micro-gripper with utilized Hoeken lineage for parallelly moving of fingers [10]

2.2 PRECISION POSITIONING MICRO-STAGES

An example of nano-manipulation mechanisms based on Scott-Russell straight-line mechanism is developed on [12] (see Figure 2 a)). Such mechanism has only one moving axis for manipulation tasks. The flexure-based Scott-Russell mechanism is monolithically constructed to provide high positioning accuracy and long-term repeatability what is obtained by piezoelectric actuator.



Figure 2 a) Nano-manipulation mechanisms based on Scott-Russell straight-line mechanism [12]; b) Design of our XY micro stage with minimized parasitic deformations based on Watt mechanism

In the design of linear motion stage with two motion axes is very important minimize cross-displacement effect caused by properties of flexure joint, because replacement of revolute joint with flexure joint is not ideal and flexural joint create undesired movement in other axis. Our design of precise motion stage based on minimal crossdisplacement effects and flexure-based Watt mechanisms is delineated on the Figure 2. b).

3 PROBLEMS IN DESIGN PROCESS OF SMALL ROBOTIC DEVICES WITH STRAIGHT-LINE STRUCTURES

The design of compliant mechanisms with lumped compliance is based on similarity with design process of classical mechanical devices [6, 7]. Between both designs procedures are small differences, and the main difference is transformation of classical kinematical pairs to compliant kinematical pairs. Two basic transformations of classical kinematical pairs with compliant element are shown on the Figure 3. For instance, transformation of turning pair is usually based on application of flexure joint, which is the connection of two relatively rigid parts with flexure element which is usually place with notches. Shape of notches defines (express) the properties of this main building element of compact compliant mechanism [3, 5]. One example, circular flexure joint has relatively linear working characteristic, but in other site maximal deformation is small, but for instance, the right-circular joint could produce larger output deformation but moving the rotation of centre is bigger. Transformation of sliding pair to compliant device is much more complicated, because in compliant mechanisms don't exist directly substitution. Such mechanisms are compounded from some flexure joints to parallelogram. But movement of parallelogram is on circle and not straight-line. This is one reason too, why it is necessary to use structures of straightline mechanisms in compliant devices for producing of exact linear motion.



Figure 3 Basic transformations of classical kinematical pairs with compliant element

Because compliant structures work on elastic deformation of material, the expression between deflection of structure (for instance of basic build block flexure hinge) and actuated force/torque is expressed as

where, on the principle of reciprocity

$$C_{yMz} = C_{\theta zFy}$$

$$C_{zMy} = C_{\theta yFz}$$
(2)

where u_i and θi are arisen deformations, matrix components C_{iLi} represents compliances, where first part of sub-index express direction of deformation (movement or rotation) and second part of sub-index express acting load (force/torque). The vector composed from F_i and M_i express common load acting to the compliant device.

Because motion of compliant mechanisms is produced by elastic deformation of used material, the replacement by flexure joint in the case of turning pair is not equal. Maximal rotation of flexure joint is depended on dimensions itself joint and mechanical properties of used material. Maximal arisen stress in the flexure joint is expressed by (3) and this value must be smaller as allowed stress (Yield strength) of used material.

$$\sigma_{\max} = K_{ta} \frac{F_x}{wt} + K_{tb} \frac{6(l_F F_y + M_z)}{wt^2}$$
(3)

where K_{ta} and K_{tb} are theoretical stress concentration factors, where K_{ta} is connected with axial load, and K_{tb} is connected with bending. Parameters w (joint width) and t (thickness – thinnest place of the flexure joint) are construction parameters and l_F is distance between joint and actuation place of force F_y . Theoretical stress concentration factors are given on the base of experimental measurements or by approximate theoretical calculation. For instance, the theoretical stress concentration factors for circular flexure hinge are

$$K_{ta} = 3.065 - 3.472 \left(\frac{2r}{2r+t}\right) + 1.009 \left(\frac{2r}{2r+t}\right)^2 + 0.405 \left(\frac{2r}{2r+t}\right)^3$$
(4)

$$K_{tb} = 3.065 - 6.637 \left(\frac{2r}{2r+t}\right) + 8.229 \left(\frac{2r}{2r+t}\right)^2 - 3.636 \left(\frac{2r}{2r+t}\right)^3$$
(5)

Other problems connected with design of compliant devices with structure of straight-line mechanisms are gained according to kinematical analysis. Using MATLAB SimMechanics toolbox kinematic analysis from chosen lineages was made. The analysed straight-line mechanisms are chosen with respect to relatively easy structure and/or according to basic research indicated in the introduction, where are described some applications of small robotic devices utilized straight-line kinematical structure in structure of whole compliant device.

Table I - Results from the kinematic analysis of some SLM

	Output motion			Joints rot. /[°]	
SLM	u _{out} /[m]	$u_{err}/[m]$	θ _{out} /[°]	min.	max.
Peaucellier– Lipkin	0.045	1.80e-9a	-	29.68	76.59
Scott Russell	0.0447	7.94e-09	180	180	180
Hoekens	0.0444	9.75e-05	58.5	26.58	209.25
Watt	0.0132	2.41e-04	18.3	52.36	55.13
Chebyshev	0.02	4.88e-05	180	53.13	126.75
Roberts	0.0456	8.96e-04	74.98	69.66	141.43

In the Table I. are presented some results from the kinematic analysis of chosen straight-line mechanisms (SLM). Table is divided to two different parts (output motion and joints rotation). The first group is connected with motion of output point of the mechanisms. Variable u_{out} express maximal movement similar to straight-line motion. Variable u_{err} express amplitude between ideal straight-line motion and real motion of analyzed device (see Figure 4). Last variable θ_{out} express the value of rotation between stable (global) coordinate system and coordinate system located on the output point. For the first analysed device is not defined value of rotation of local coordinate system located at output point of lineage to global coordinate system because, in this case the output point is revolute joint. One conclusion from data analysis is that extract straight-line motion mechanisms produce better straight-line motion as approximate devices, where

waviness is around hundred micrometers compared to units of nanometres.

Second group of values in the Table I are connected with rotation of revolute joints of analysed device in process of producing output straight-line motion. According to values of angles it is clear, that classical kinematic structures of straight-line mechanisms utilize revolute joint as much as it is possible, but in the case of compliant structures the rotation of flexure joint is limited by material properties, by dimensions and by form (shape) of notch.

Other problem of such devices is rotation of output point towards to global coordinate system (θ_{out}). This problem is in the case of compact compliant mechanisms design solved by two main approaches:

- adding a sliding pair,
- adding parallel mechanism.

In the case of adding a sliding pair to minimize rotation of output point is approach based on, adding to output point of device one revolute joint and connect it with moving part of sliding pair. This approach is for instance presented on [13], were some straight-line mechanisms (see mechanisms labelled as S1, S3, S12, S36 and others) utilize sliding pair for produce straight-line motion.

Other approach is based on utilization of equal parallel mechanisms, where again at output point of device is added revolute joint, and both joints (original and from added device) are connected by relatively rigid element [14]. In case that both mechanisms are actuated synchronous, or in this same place by one actuator, then it is not possible to gain crossing. Similar approach was utilized in the case of described micro-gripper (see Figure 1) where whole Hoeken lineage was not added to the structure, but only parallelogram.



Figure 4 Output trajectory and detail of some analyzed straight-line mechanisms

4 COMPLIANT PEAUCELLIER-LIPKIN MECHANISM

The Peaucellier-Lipkin mechanism was the first planar straight-line mechanisms capable transforming rotary motion into perfect straight-line motion [15]. The main reason, why it is necessary to transform such devices intro compliant mechanism is, that many straight-line structures

utilized in compliant devices belong into group of approximate straight-line mechanisms (see Figure 1 and 2). What mean, that with transformation to the compliant device we could not get ideal straight-line output motion, but only their approximate equivalent. But in the case of the compliant devices, with the parasitic deformations we must calculate to the output motion, what mean, that output motion of compliant device, is combination of error gained from parasitic deflections and error arise from approximate straight-line device. In the case of Peaucellier-Lipkin mechanism, the error u_{err} (see Table I) is very small and therefore whole error of output trajectory from compliant device is combination of parasitic deformations and changing of the structure (see below).

The mechanism consists of isosceles four bar chain *OCBD* (Figure 5). Additional links *AC* and *AD* from, a rhombus *ACBD*. *A* is constrained to move on a circular path by the radius bar O_1A which is equal to the length of the fixed link OO_1 [16].



Figure 5 The Peaucellier-Lipkin straight-line mechanism

From the geometry of the figure, it follows that AC = CB = BD = DA

$$OC = OD; \quad OO_1 = O_1A \tag{6}$$

It may be proved that the product $OA \times OB$ remains constant, when the link O_IA rotates. Join CD to bisect AB at R. Now from right angled triangles ORC and BRC, we have $OC^2 = OR^2 + RC^2$ (7)

$$BC^2 = RB^2 + RC^2 \tag{8}$$

Subtracting (8) from (7) $OC^2 - BC^2 = OR^2 - PR^2$

$$= (OR + RB)(OR - RB)$$

$$= OB \times OA$$
(9)

Since OC and BC are of constant length, therefore the product $OB \times OA$ remains constant. Hence the point *B* traces a straight path perpendicular to the diameter OP.

4.1 KINEMATICAL ANALYSIS

As was mentioned, during design of compliant mechanisms with lumped compliance it is necessary to transform kinematic structure of designed device to flexure structure. However, the kinematical structure of Peaucellier-Lipkin mechanism use revolute joints, which connect three different rigid bodies (see Figure 5. points A, C, D), but as is clear from the Figure 3., flexure joint can connect only two relatively rigid bodies with elastic element. Therefore the kinematical analysis of some modifications such device was made for gaining structure which is build from revolute joint which connect only two rigid parts. Other problem is, that we will use designed mechanism as one building part of other devices, for instance precise positioning micro-stage, therefore it is necessary to design such device, where straight-line output motion will not be produced by a point but minimal one edge. All kinematical analysis was made in MATLAB SimMechanics toolbox.



Figure 6 a) Kinematic model of Peaucellier-Lipkin straight line mechanism in MATLAB SimMechanic toolbox, b) trajectory of output point of analyzed PL SLM

Kinematic analysis of some modifications of Peaucellier-Lipkin mechanism was analysed. All modifications were made with respect to the pole of planar displacement, which is that point which does not move under an arbitrary planar operator. It is unique, unless the planar operator is a pure translation. An operator can be related to a different frame by pre- and post-multiplying by the coordinate transformation to the different frame and its inverse respectively. When related to the pole, the equivalent operator is a pure rotation [17].



Figure 7 Approaches how to transform joint in the point A, a) horizontal, b) vertical

Two approaches were analysed in the case of transformation of joint located at the point A. First one is

based on horizontal movement and dividing of such joint (see Figure 7a)) and second approach was based on vertical movement and dividing of joint located on the point A (see Figure 7b)).

According to comparison of output movement of both structures the first approach based on horizontal movement of joints was completely removed from the further analysis. In the case of transformation of the joint located on the point C or D, more approaches had been studied. The link OC (OD) can be connected to the link AC or BC, and in both cases minimum of three types of connections are possible. On the Figure 8 are shown all analysed modifications of joint C.

The results from the kinematic analysis of the modifications of Peaucellier-Lipkin mechanism are presented on the Table II.

Table II - Results from the kinematic analysis of modified Peaucellier-Lipkin mechanism

of modified readeciner-Lipkin meenanism					
Modification	u _{err} / [m]	$\theta_{out} / [\circ]$			
mod_01a	47.04e-5				
mod_01b	7.00e-05	9.86			
mod_02a	1.46e-04	11.38			
mod_02b	4.63e-04	9.86			
mod_03a	1.58e-03	2.232			
mod_03b	1.44e-03	3.3			
mod_03c	1.52e-04	8.84			
mod_04a	2.24e-04	9.86			
mod_04b	3.79e-04	9.86			
mod 04c	1.17e-04	9.86			

Considering to results from kinematic analysis of modified Peaucellier-Lipkin mechanism only two modifications (mod_02a and mod_04c) will be analysed below. The reason of choice is that such devices have smallest error in relationship to ideal straight-line motion, in other site, rotation of output edge is higher as in modification 3b and 3c, but the error in such modification is relatively high. In the Table II are modifications called mod_01a and mod_01b, such modifications are shown on the Figure 7, but in the case of mod_01a was added parallelogram to the link *CB*.



4.2 FEM ANALYSIS

For chosen kinematic structures of modified Peaucellier-Lipkin mechanism was designed compliant structures. The Comsol Multiphysics software was used as tool for design verifying and as tool replaced experimental results. In the Figure 9 are drawn compliant structures of analysed devices.

Input parameters were: material Polyamide PA2200 with Young's modulus 1.65GPa and with Yield strength 48MPa. The width of structures was 40mm and height around 20mm; thickness of both structures was 1mm. Dimensions of flexure joint; radius 0.525mm, and joint thickness 0.175mm. Because we want to know relatively real results of mechanisms behaviour nonlinear solver was applied, in the case of linear solver the results were relatively good, when the error u_{err} was around between 5e-9 to 3.08e-7 (see Figure 9c) and 9d)), but as it is clear from the kinematic analysis (see Table II.) such values was expected in different range of values.



Figure 9 Compliant structures of modified Peaucellier-Lipkin mechanism a) mod_02a; b) Stress and deformation of compliant Peaucellier-Lipkin mechanism (modification mod_04c), Trajectory of output point of compliant PL SLM from FEM analysis c) linear solver, d) non-linear solver

In the Figure 9 b) it is shown how the compliant structure (mod_04c) is deformed and where the maximal stress is arisen. As is clear from this figure, the stress is concentrated in the flexure joint of small parallelogram; the reason of this is that on small parallelogram load is acted. The Figure 10 showed the results from the FEM analysis of chosen compliant structures of modified Peaucellier-Lipkin mechanisms. On the figure the output trajectories are shown, last trajectory is from the FEM analysis of

parallelogram, with equal dimensions. According to results it is clear that our design of compliant straight-line mechanisms has better properties as application of the parallelogram. On the other site our structure is more complex as easy parallelograms structure.

The numerical values of results from FEM analysis are:

- Mod_02a: $u_{err}=15.5\mu m$, $\theta_{out}=4.415^{\circ}$
- Mod_04c: u_{err}=5.92μm, θ_{out}=4.608°
- Parallelogram: $u_{err}=37.9\mu m$, $\theta_{out}=0.027^{\circ}$



5 CONCLUSION

Structures of straight-line mechanisms are very often used in building of smart precise robotic devices, to which requirements based on high accuracy and precision with manipulation of micro- and nano- scale objects are put. The paper deals with problems in design process of one specific straight-line mechanism worked on principle of compact compliant mechanical structure. It was shown that theory about the pole of planar displacement could be utilized for transformation of revolute joint which connected more than two rigid bodies to group of joints which connect only two rigid bodies, what is necessary condition to design of compliant structure. The kinematic analysis of some modified Peaucellier-Lipkin straight-line mechanisms was made in MATLAB SimMechanics toolbox and after results evaluation only two modifications had been transformed to the compact compliant structure. The FEM analysis is confirmed rightness of both mechanisms design. The movement of the designed compliant straight-line mechanism is six-times better and closer to the straight-line motion as it is in the case of the parallelogram. In other site, the problem of rotation of output point/edge in values around plus/minus 2.3 degrees is not acceptable now.

In the future work the combination of advantages of designed straight-line mechanism and parallelogram should be done. The simple parallelograms structure and precision of compliant straight-line mechanism motion could be one way how to design compliant multi-axis positioning micro-stage whit small requirements to the control system, and with minimized parasitic deformations.

ACKNOWLEDGMENT

This paper presents the research work supported by the national scientific grant agency VEGA under project No.: 2/0048/13 "(Micro) Electro-mechanisms for robotics and extremely work spaces (environments)."

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