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STOCHASTIC MODELLING AND EXPERIMENTAL OUTCOMES OF MODAL ANALYSIS ON AUTOMOTIVE WHEELS

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ABSTRACT

An experimental investigation of the effects of the fitting procedure of disk and rim in automotive wheels on their dynamic behaviour is reported. Due to the variation of mass and fitting diameters of the two components, the goodness of the process is evaluated through the assembled wheel dynamics. The geometrical data of an experimental measuring campaign on a large sample of each sub-component are provided and critically analysed with statistical methods, while the results of an experimental modal analysis supply the modal properties of each component. The potential of this method lies in the possibility of being able to deduct the residual stress due to the fit, difficult to quantify in other way, directly from the correlation of the experimental measurements of the natural frequencies of the sub-components and of the wheel after the fitting. Finally, an indirect measure of the fitting procedure can be detected.

Keywords: experimental modal analysis, interference fit, automotive wheel

1 INTRODUCTION

The search for more and more performing and safe vehicles has led, in recent years, to an increasing optimization of all automotive components.
Components designed are safer at the same time lighter to comply with emissions standards become progressively ever more restrictive.
Nowadays the dynamic behaviour of automotive wheels has become particularly important for the overall performance of the vehicle. The design of these components must be accurate enough because they have a particular importance and on their dynamic depend driving stability, safety and most of the vibrational and acoustic comfort of the vehicle. In order to satisfy these aims, the study of the wheel should include analytical and numerical models to take into account the quasi-random variations in the real components due to the production process.

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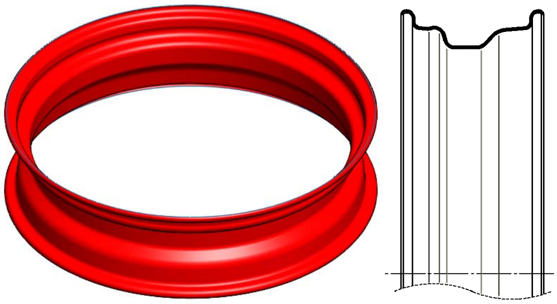
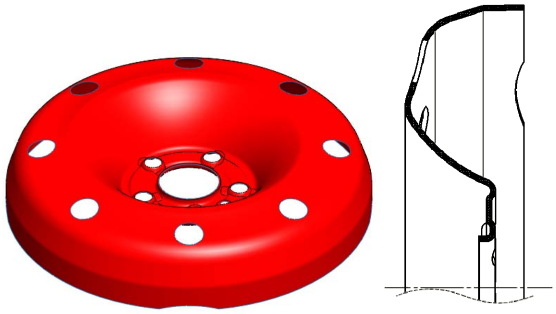
The stochastic changes of the main variables involved, owing to the dimensional and geometric tolerances, assume a major role. A large part of the automotive wheels are obtained by fitting the disk in the rim: this process produces stress on the component, in addition to the residual production process residual stress, that is very difficult to estimate due to variability in the production process and due the geometric and dimensional tolerances of the components.
Several studies have been done on the wheels classical fatigue tests: in [1] a simulation of the cornering fatigue test is provided, in [2] the radial fatigue test of the wheels is presented, but in both cases the effects of the fitting are not considered.
The determination of the service stress of a wheel, due to the vertical and horizontal loads and production process is reported in [3], but also in this case the residual stresses are not considered.
In [4] the authors also investigated the effect of residual stresses due to production process and the fit of the disk in the rim, considering a large sample of disks, rims and wheels.
The stress causes variations in the frequencies of the components, stress stiffening or softening effect: in [5] is shown as the stress affecting the dynamic behaviour of the

component, while in [6] an analytical formulation is obtained for the fundamental frequency for a clamped plate increasing stress.
The fitting of the disk in the rim produces plastic deformation of the components. Therefore it is not possible to deal the wheel with linear elastic equations; recent works show the limitations of the classical thick-wall theory when dealing with complex geometry components and with plastic deformations [7].
It is well known that any measure made on nominally equal components has some variability [4, 8, 9, 10], so it is not possible to estimate in advance the stress due to fitting because it is influenced by a large number factors, and because the stress on the wheel depends on the two particular component assembled.
The basic idea of this work is that of being able to estimate the tension due to the fitting of disk and rim, just starting from the measurement of the natural frequencies of the components and subsequently of the assembled wheel: in effect, at each modeshape of the wheel is possible to associate a component modeshape [4].
A statistical approach is needed to study the influence of the components properties on their natural frequencies, this paper in particular focuses on the effects of mass and fitting diameter. To correlate fitting diameter and mass to the natural frequencies, an experimental modal analysis campaign has been conducted on a sample of 40 disks, 40 rims to detect which of the modeshapes is useful to the aim.



(a)



(b)

Figure 1 Isometric view (left) and cross section (right) of the rim (a) and disk (b) considered.

2 MEASURED DIAMETERS AND MASSES

The subject of the study is a spare wheel of a common car, with the disk and rim shapes and cross section shown in Figure 1.
The fitting diameter considered is the minimum in case of the rim, and the maximum in case of the disk, thus the ideal interference value is the difference of the two.
While the 40 disks come from the same component specifications and thus they only differ one another for production tolerance ranges, the 40 rims are divided into 3 families:

- standard rims (20 samples);
- increased diameter rims (10 samples);
- decreased diameter rims (10 samples).

With those families in addition to the standard interference fit range, two extreme ranges can be taken into account, one with very low and the other with very high interference fit. The residual stresses caused by the interference conditions after the fitting process are presented in Figure 2.
The masses and the diameters for the samples of disks and rims are subject of a first statistical check using Normal Probability Plots (NPP) to evaluate the normality of the distributions.
Figure 3 shows the distributions of rims and disks diameters, where in the top plot the three rim families are

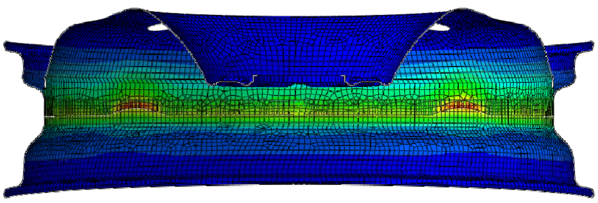


Figure 2 Contour plot of the residual stresses due to the fitting process, from numerical simulation.

clearly distinct, and in the bottom plot the disks have an unexpected trend in the central zone, raising the possibility of a bi-normal distribution.
One of the possible causes of this distribution might be due to different sheet metal coils.
The same plots are shown in Figure 4, considering the mass property. Each component was measured four times and randomised with other samples, and in this plots all the single measurements are represented.
In both plots a gap is present, dividing the range into two separate distributions. In both cases the trends are hypo-normal, with several samples located at the tails of the distribution, and considering the left portion the trend is more normal with respect to the right portion for both rim and disk cases.

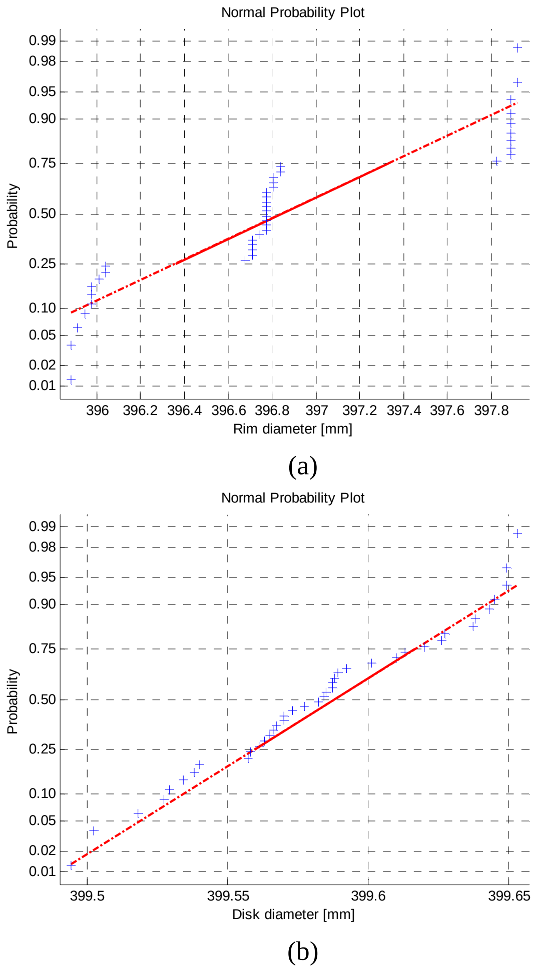


Figure 3 NPP for rims (a) and disks (b) diameter.

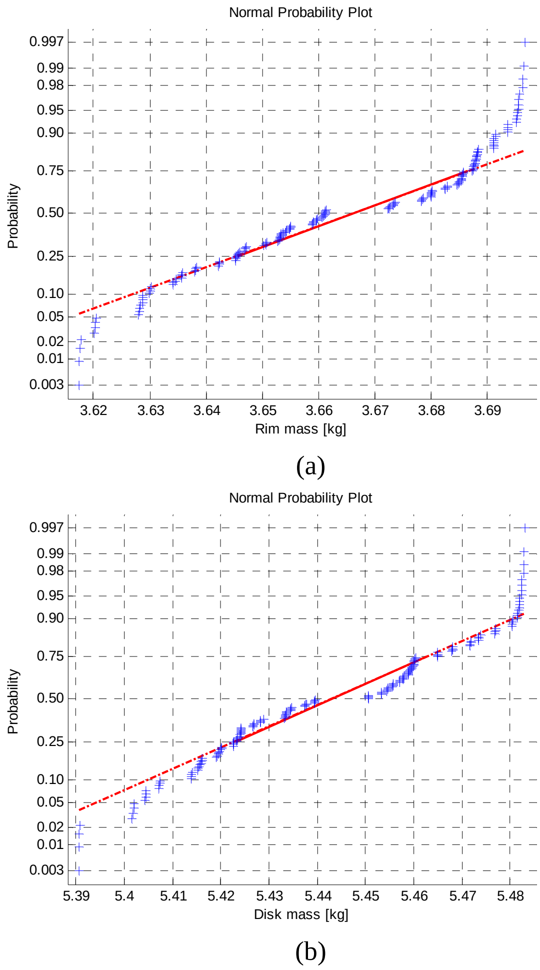


Figure 4 NPP for rims (a) and disks (b) mass.

Analysing the rims, the mass NPP can be arranged considering the averages of the four measurements of each component, and colouring differently the three diameter families, as shown in Figure 5. It is interesting to notice that while standard and bigger rims distribution are mixed, with bigger rims slightly heavier, the smaller rims are more concentrated and located on higher values. This says that smaller rims are heavier.

Considering the sheet metal thickness as one of the most relevant parameter which influences the mass, the heavier smaller rims could be due to the non-standard production process of the component, which could induce a lower influence on the thickness.

3 EXPERIMENTAL MODAL ANALYSIS

For the correlation analysis, Experimental Modal Analysis (EMA) has been performed on each sample of disk and rim, in free-free boundary conditions simulated using low stiffness springs. The data are acquired using two monoaxial accelerometers, placed in three different configurations on the rims and two configurations on the disks, and the excitation chosen is impact hit with force detection hammer.

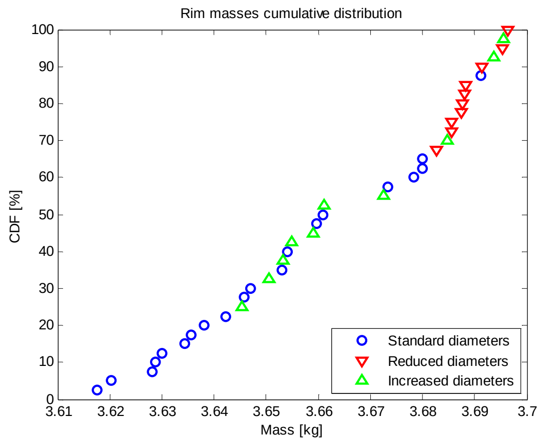


Figure 5 NPP for rims mass in separated families.

The aim is not to perform a full modal identification but only the Frequency Response Function (FRF) in order to identify the natural frequencies of each component, not considering the whole modeshapes.

Figure 4 and Figure 5 show the acquired FRFs for one sample of rim and one sample of disk respectively, where the different configurations for disk and rim are clear.

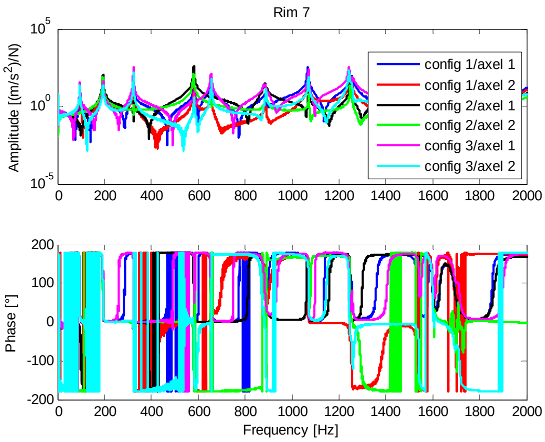


Figure 6 FRFs for rim #7.

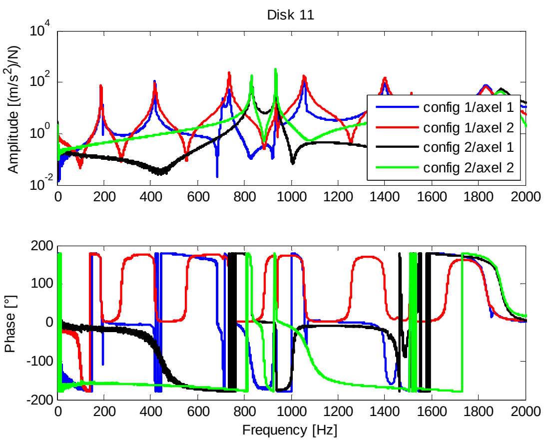


Figure 7 FRFs for disk #11.

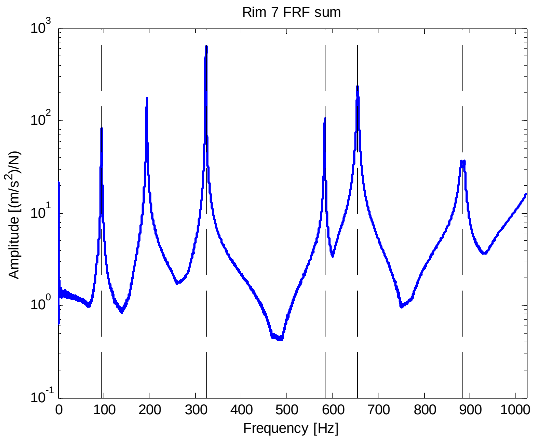


Figure 9 FRFs sum and frequencies for rim #7.

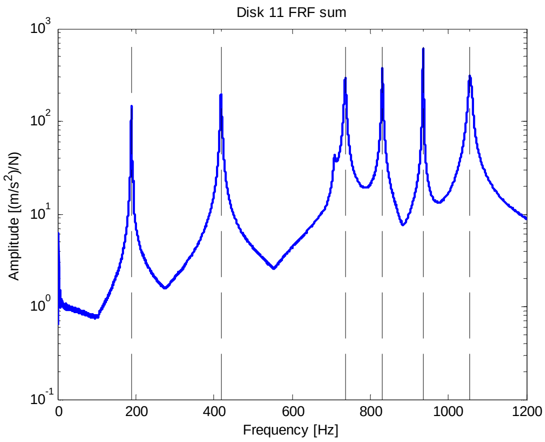


Figure 10 FRFs sum and frequencies for disk #11.

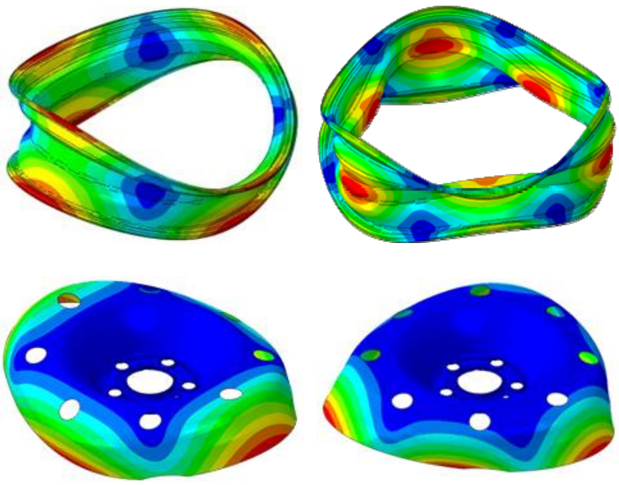


Figure 8 Numerical modeshapes of rim (upper) and disk (bottom) for the 1st mode (left) and the 4th mode (right).

Knowing the location of the accelerometers, it is possible to determine the modeshape, known the modeshapes sequence from a numerical simulation. Figure 8 shows the 1st and the 4th numerical modeshapes of disk and rim.

The single FRFs allow evaluating the zones where structural nodes apply by curve comparison, and helping recognising the modal shape.

In the rim case, the accelerometers for the second configuration were located in the central zone. The central zone is less relevant and it is subject to several structural nodes, thus for all the rims it has not been considered.

All the remaining FRFs have been subject to a sum, in order of obtaining a single curve FRF sum which could represent the component.

This sum was then subject of automatic peak detection for the identification of the natural frequencies, which are then assigned to each disk and rim.

Figure 9 and Figure 10 show the sum of the samples related to the single FRFs, with vertical dashed lines which represent the location of the natural frequencies identified though the automatic routine.

Due to the difficulty of automatically separate in the experimental identification the different modes when they have very similar frequency (common feature of axisymmetric objects), they are considered as single modes, thus with the same mode number.

4 FREQUENCY CORRELATIONS

The automatically identified natural frequencies for rims and disks can then be correlated to diameters and masses through qualitative evaluations on simple plots.

In the rim case, it is possible to analytically predict the dependency of natural frequencies with respect to the geometrical parameters by taking into account the dynamic bending behaviour of a generic *r*-mode, with the assumption of an equivalent curved beam with rectangular cross-section. With modal coordinates the related frequency would be

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} \qquad (1)$$

and by making explicit the stiffness *k* and mass *m* terms of the related geometry

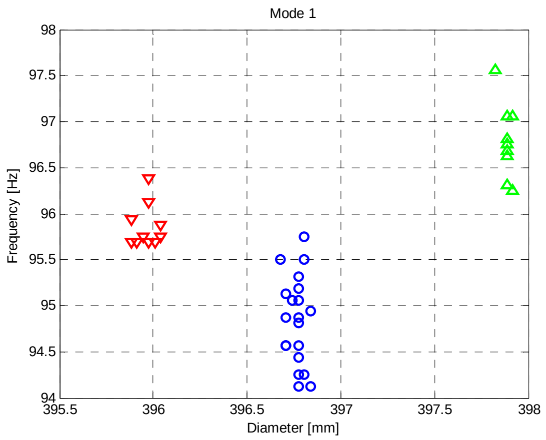
$$f_r \propto \frac{1}{2\pi}\sqrt{\frac{\frac{12\,E\,I}{l^3}}{\rho\,A\,l}} \qquad (2)$$

it is possible to show the dependency with respect to thickness *s* and diameter *d*

$$f_r \propto \frac{1}{2\pi}\frac{s}{\pi^2 d^2}\sqrt{\frac{E}{\rho}} \qquad (3)$$

with linear direct effect of the thickness and inverse quadratic effect of the diameter. For the thickness in particular the behaviour is the opposite of what one would expect: an increase of thickness would increase the mass thus lower the frequency, but in this case the stiffness contribution is predominant, providing a direct dependency between natural frequencies and mass. Similar dependency can be detected for disks, considering equivalent bending modes of a circular plate.

Figure 11 shows the correlation between the first and fourth mode of the rims, with respect to the diameters. The rim families are clearly separated and represented with different colours.



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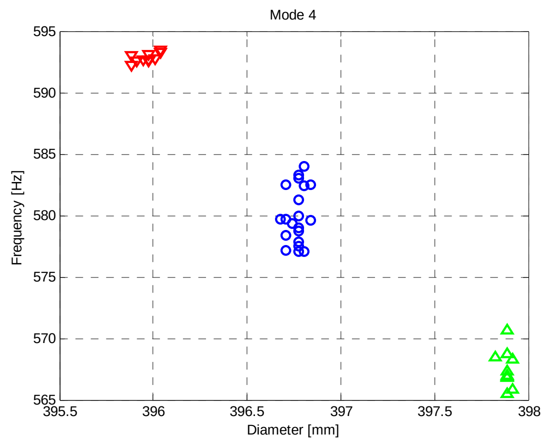
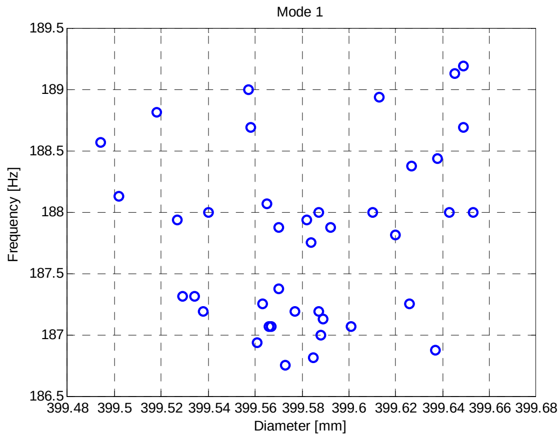


(b)

Figure 11 Frequency vs. diameter plot of rims for mode #1 (a) and mode #4 (b).



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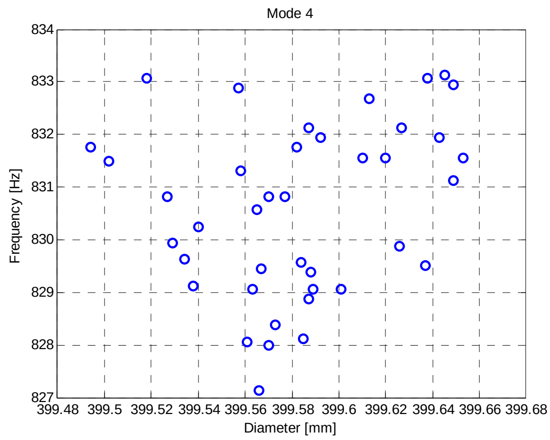


(b)

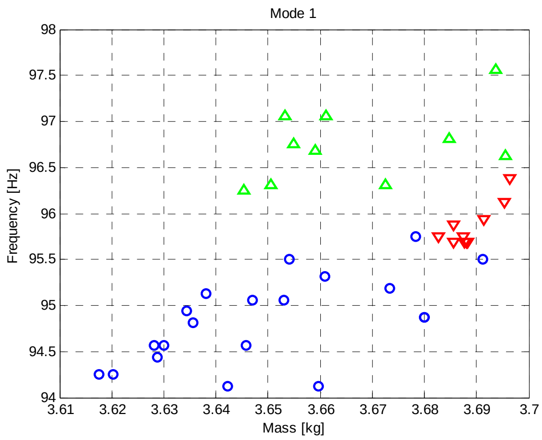
Figure 12 Frequency vs. diameter plot of disks for mode #1 (a) and mode #4 (b).

There is a general tendency, considering the difference of the families, while inside the families there is no tendency evidenced. In particular, for mode #4, the trend is a decreasing frequency on increasing diameter, as predicted in (3). Not evident tendency are detected inside each families, most probably because other parameters have the same influence on the frequency with respect to the diameter and mask the trend.
On the other hand Figure 13 and Figure 14 show the dependency of the frequencies with respect to the mass for both rims and disks respectively.
In this case the mass has particular influence on the resonance frequency and with a linear positive trend.
Figure 12 then shows the same plot but considering the disks. Having only a single standard family the plotted point are randomly scattered, it is clearer that there is no particular trend considering just the tolerance range for the diameter, as happen in each family of the rims.
Considering the rims, the trend is increasing frequency on increasing mass, and this can be counterintuitive. There are visible three different linear correlation, one for each family, depending on the diameter, as obtained before.

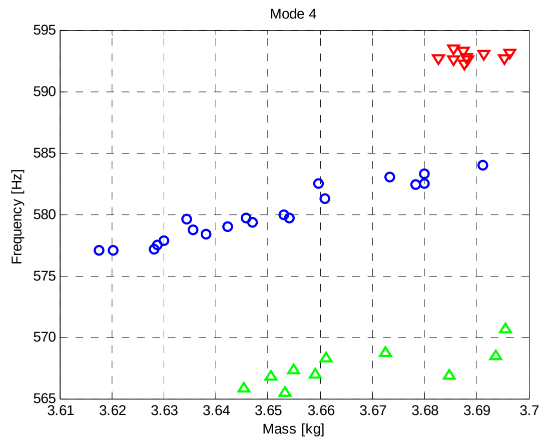
This time is also visible a trend inside the families, with much less dispersion in case of mode #4.
In the disks case, where a single family is present, the trend is still clear, and also in this case mode #4 shows a clear trend with much less dispersion. These trends tend to confirm the hypothesis of the relevant influence of the sheet metal thickness on the mass of the component, as in (3) for both rims and disks.

5 CONCLUSIONS

In this paper large samples of rims and disks for automotive wheels are been analysed. The fitting diameters and the masses of rims and disks are been measured. From statistical analysis of the diameters the three known families of rims appears clearly, but also the disks seems divided into two families probably because of different sheet metal coil. The statistical analysis on the masses evidence a strange behaviour in the rims, indeed the smaller rims results to be heavier, while the masses of standard and large rims are mixed respect the diameters.

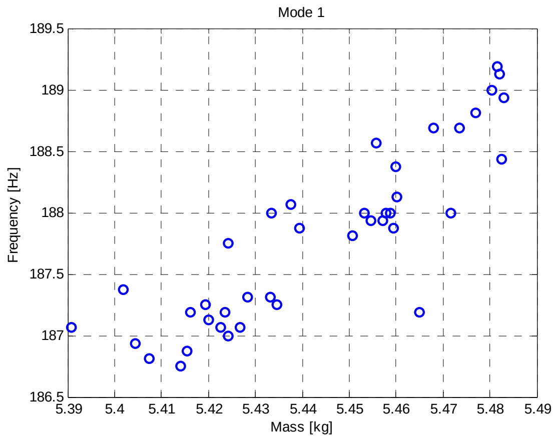


(a)

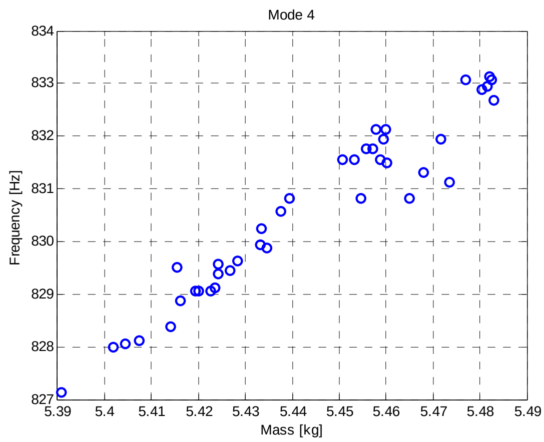


(b)

Figure 13 Frequency vs. mass plot of rims for mode #1 (a) and mode #4 (b).



(a)



(b)

Figure 14 Frequency vs. mass plot of disks for mode #1 (a) and mode #4 (b).

After the detection of the natural frequencies of each sample of rims and disks the relationship between natural frequencies and mass or fitting diameters has been researched.
The correlation between diameters and resonance frequency of the rims evidence a trend among the families but not inside each one, while the same correlation on the disks does not show any particular relationship.
On the other hand the correlation between masses and natural frequencies is clearly visible both for rims, also inside the three families, and disks. The aim is to find a modeshape for which there is a relationship between fitting diameters and its natural frequency, both for disks and rims. The fourth mode of rims and disks can be a valid candidate. The study of the corresponded system modeshape, correlated to the chosen component modeshape, can give very useful information of the stress due to the interference level.
The natural continuation of this work will be the fitting of rims and disks, in order to have as much as possible different level of interference, and studying the changes in the dynamic behaviour of the system to find the correlation with the residual stress.

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